“Local” and “Global” in terms of mathematics

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Is mathematics useful?

Do you like mathematics?

Is mathematics useful?

Do you like mathematics?

Why do we study mathematics?

Is mathematics useful?

Does mathematics help you solving (regional) problems?

Is mathematics useful?

Does mathematics help you solving (regional) problems?

No! (at least directly.)

Is mathematics useful?

Does mathematics help you solving (regional) problems?

No! (at least directly.)

But

- Mathematics lies at the base of various studies.
- Mathematics provides new viewpoints.

Mathematics lies at the base

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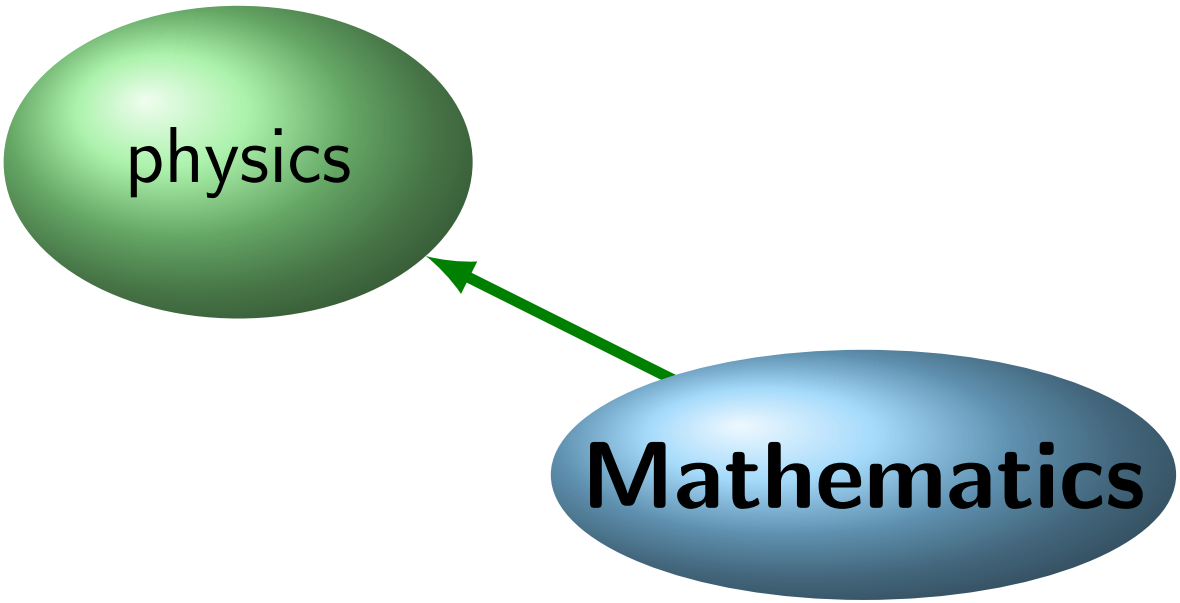

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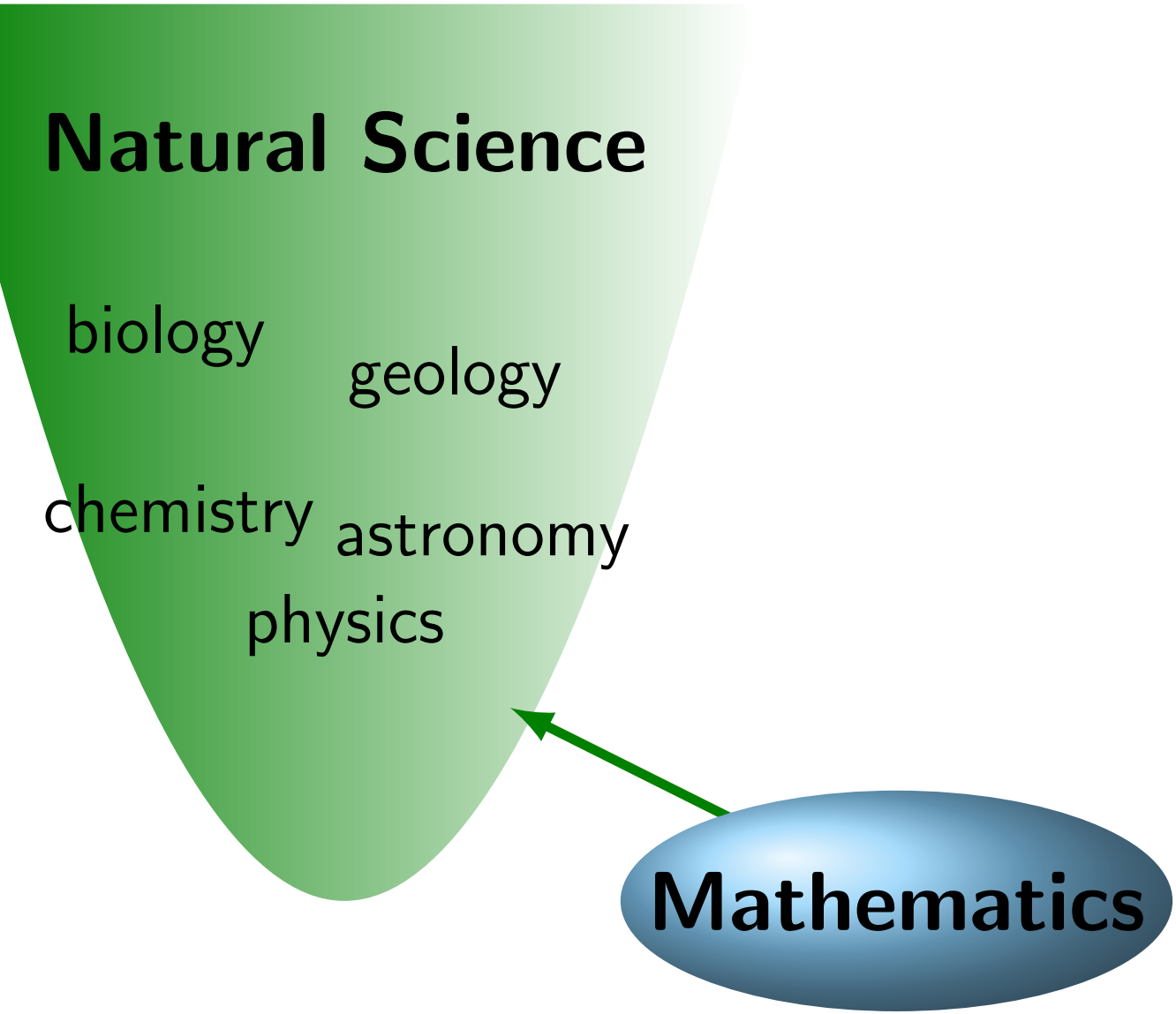

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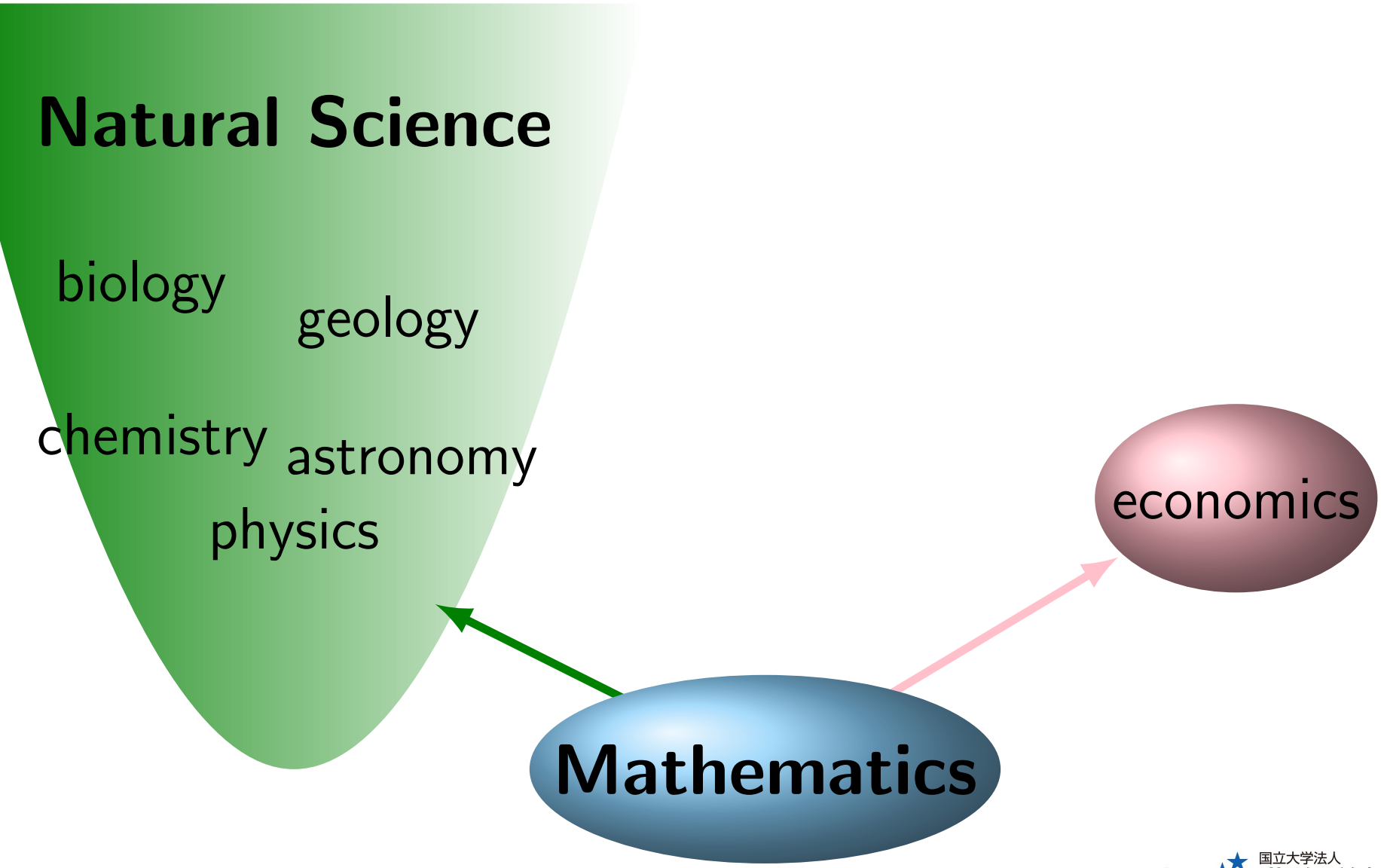

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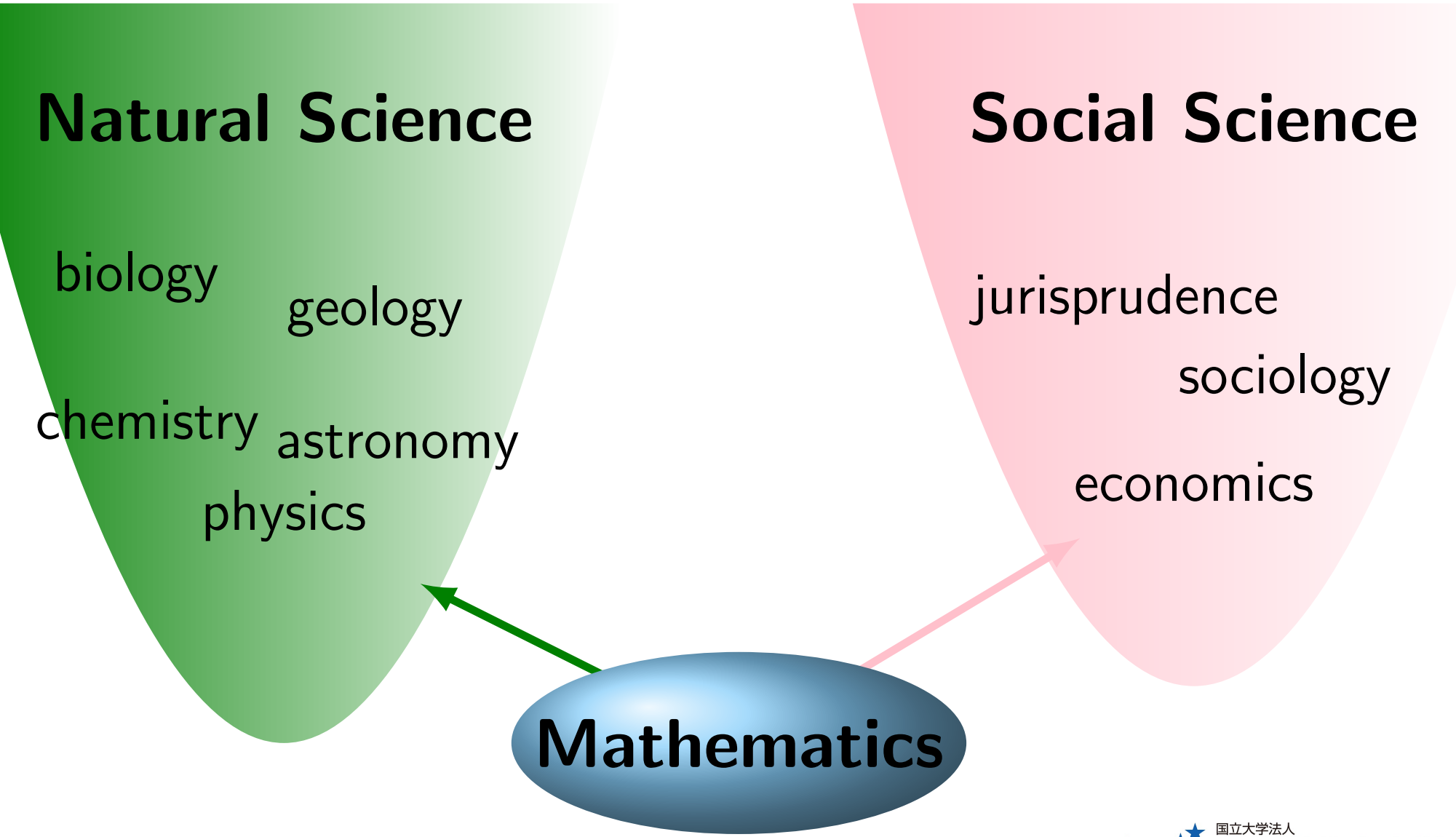

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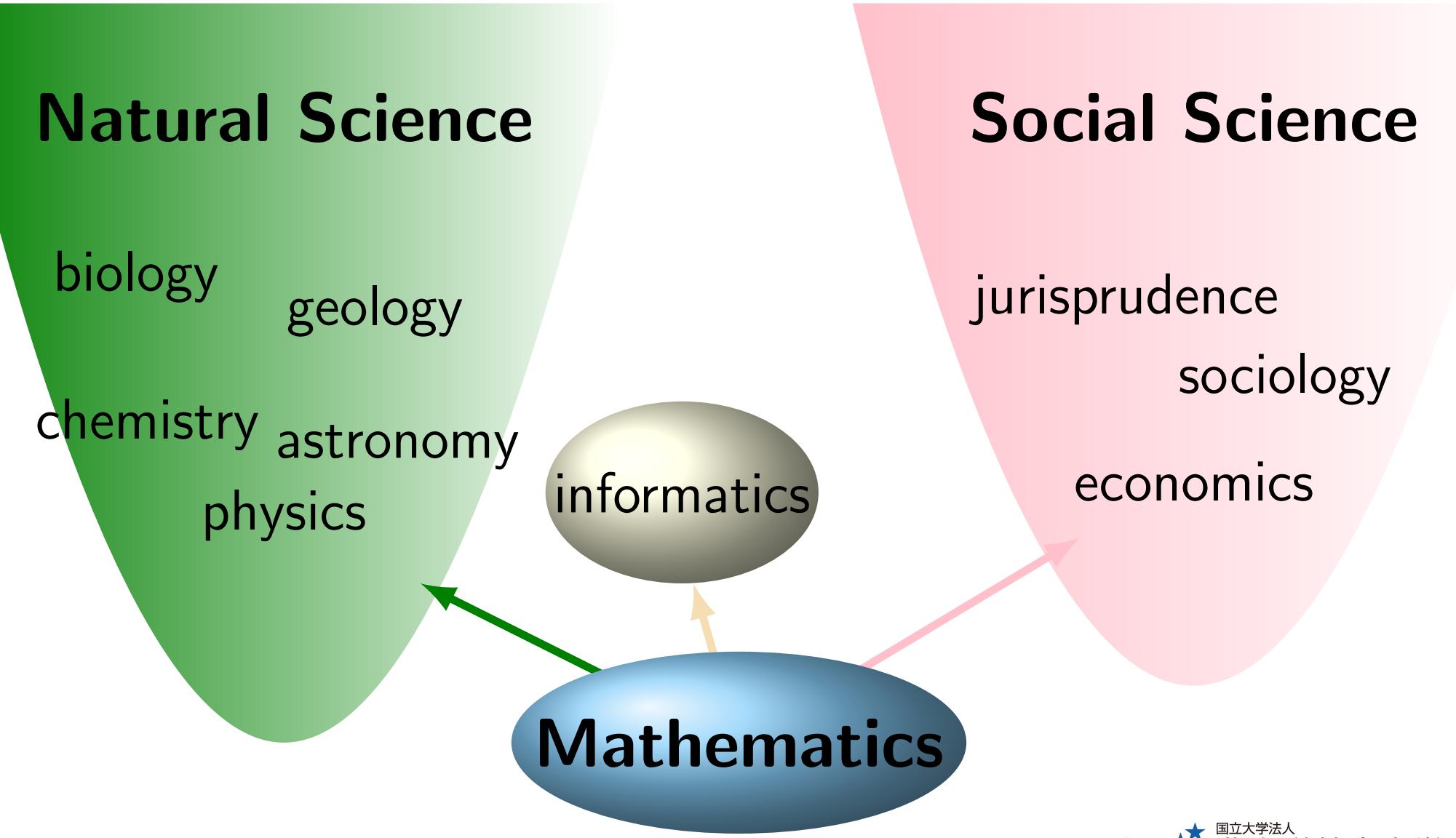

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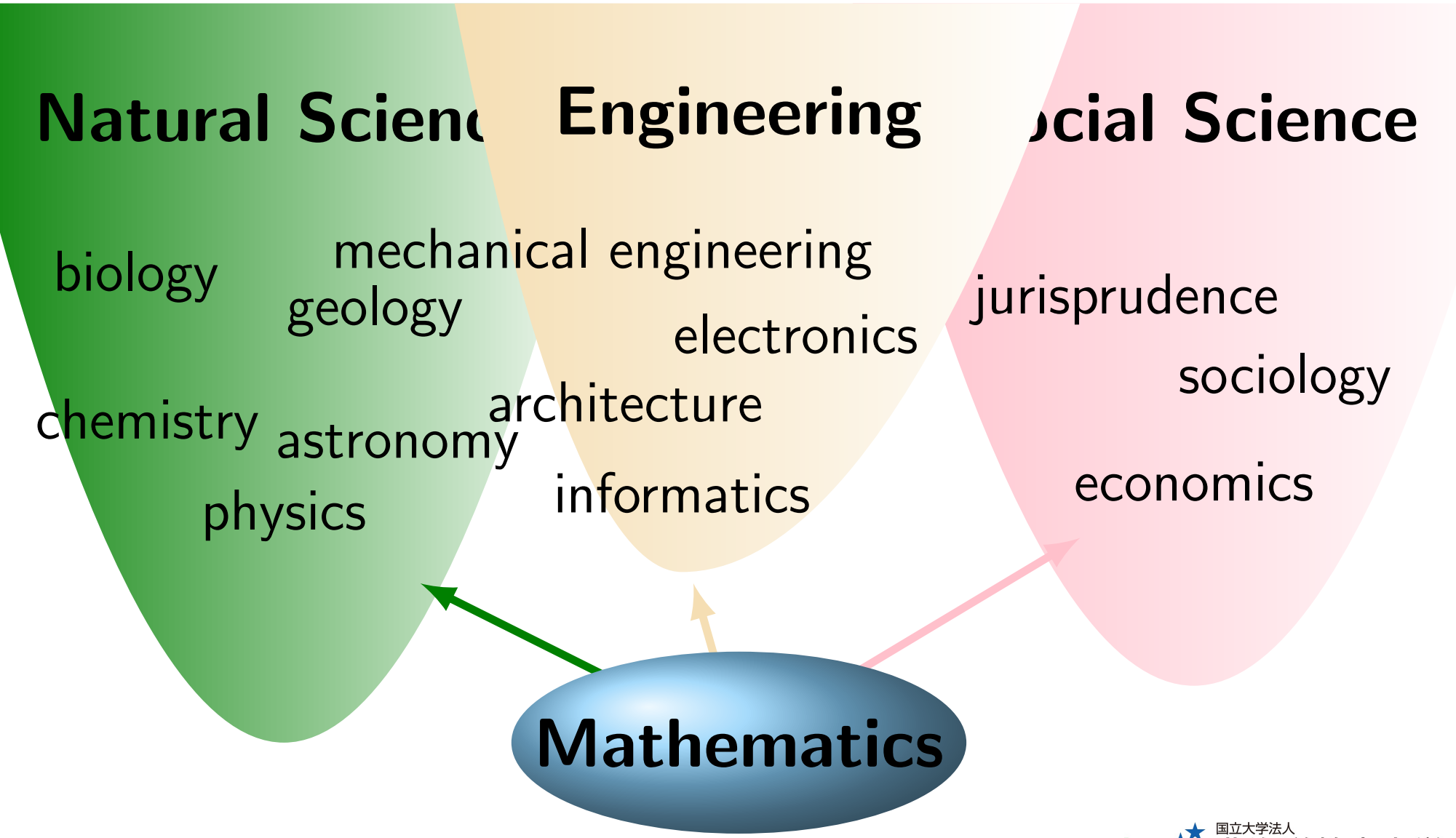

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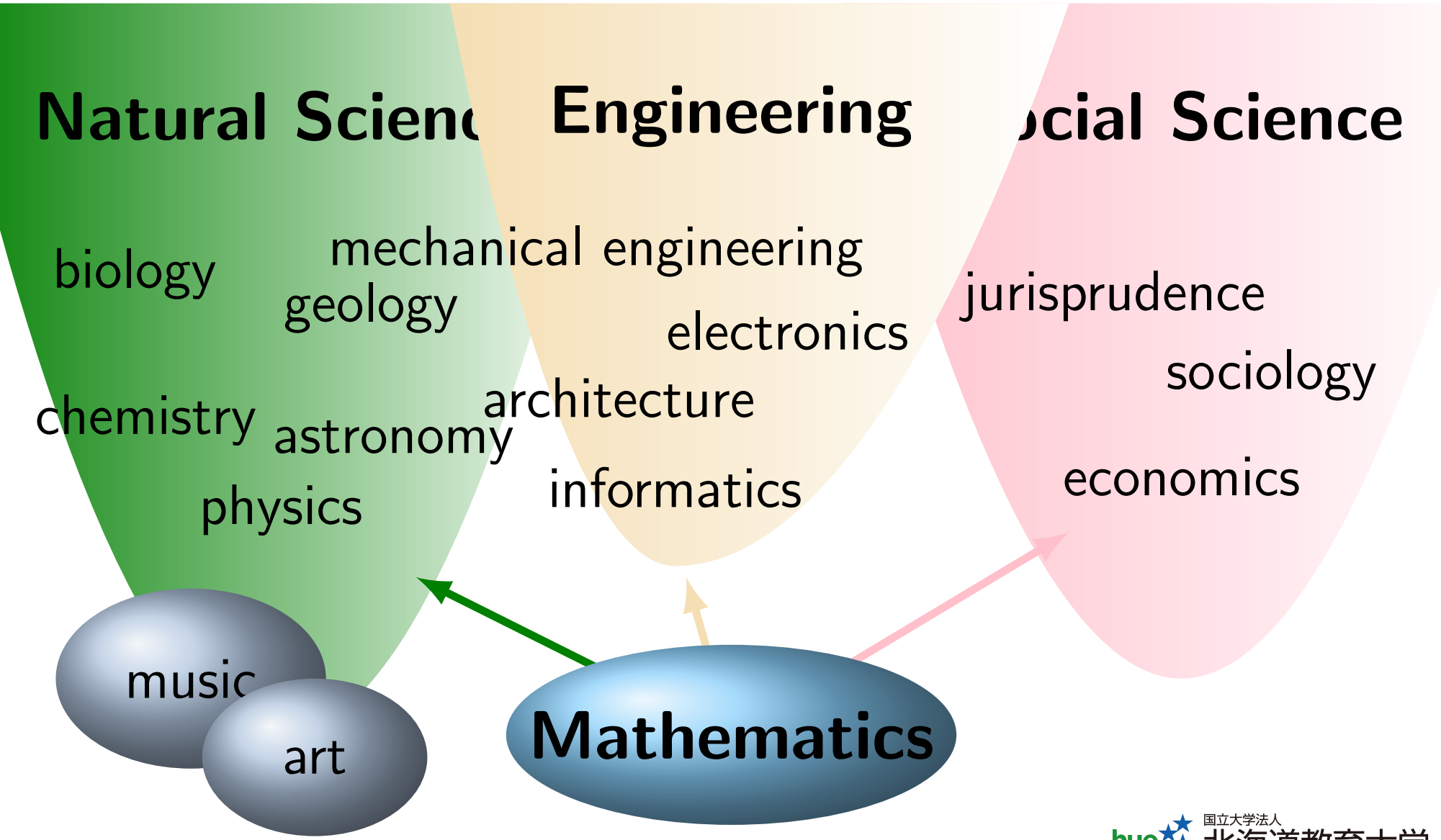

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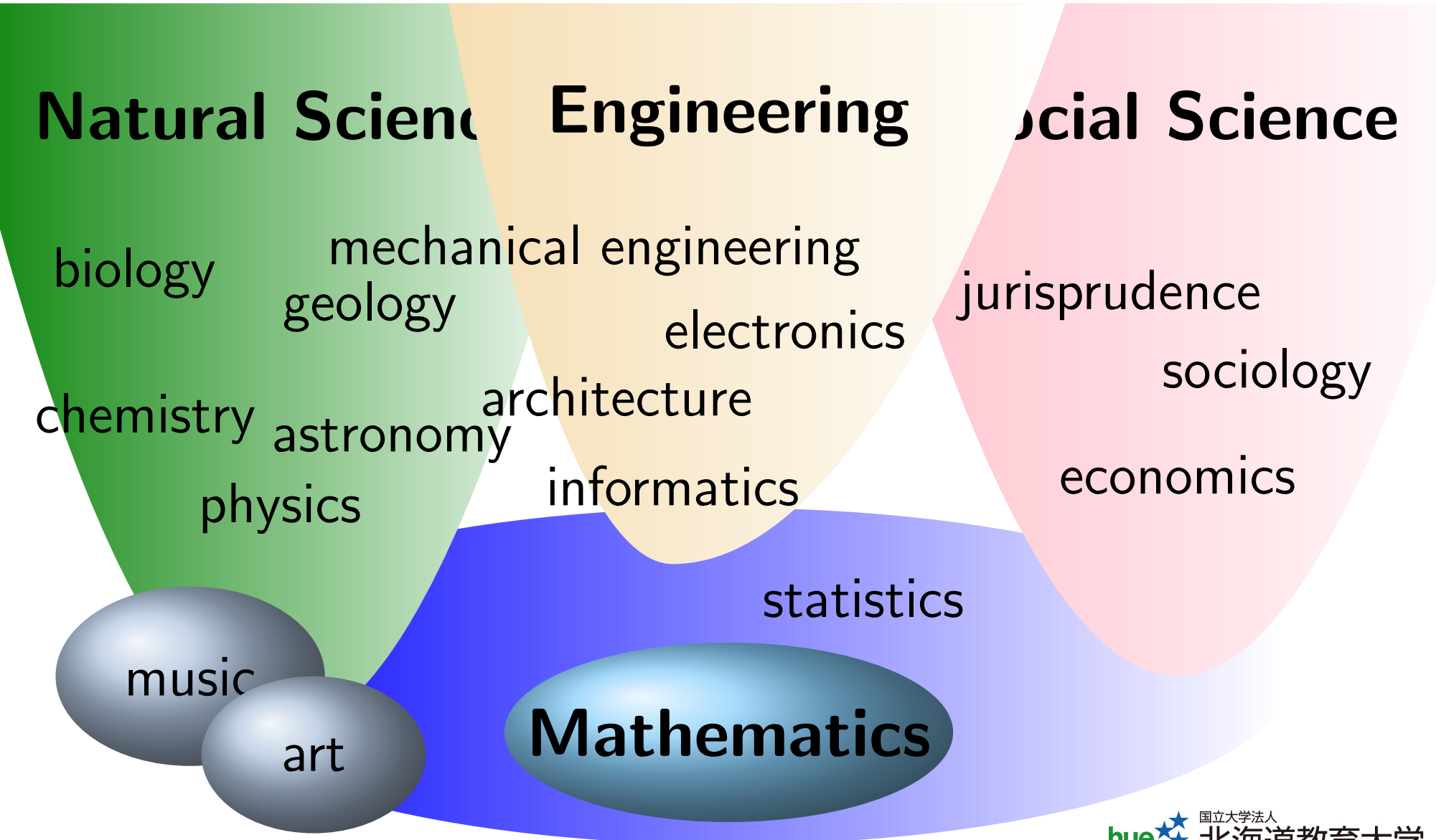

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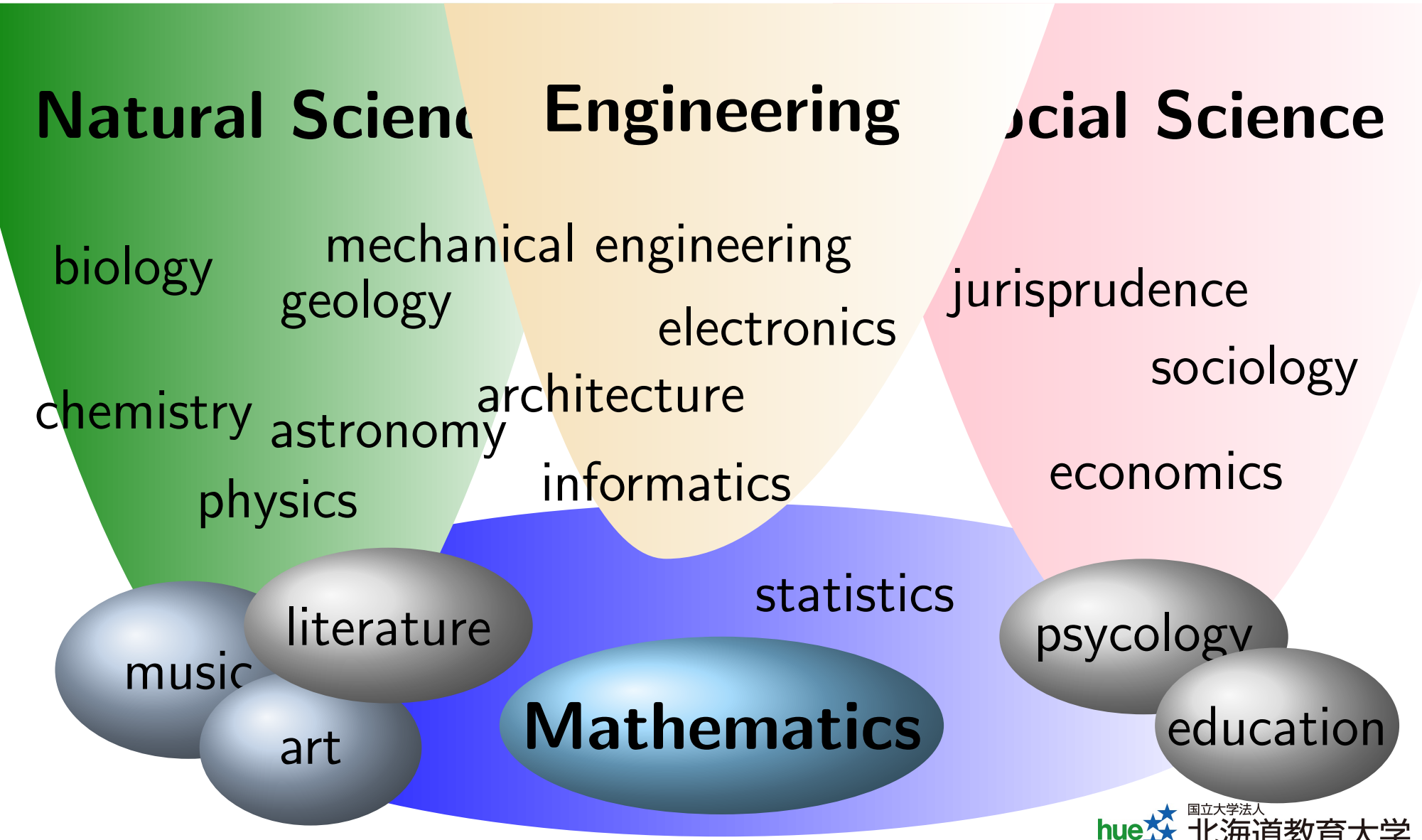

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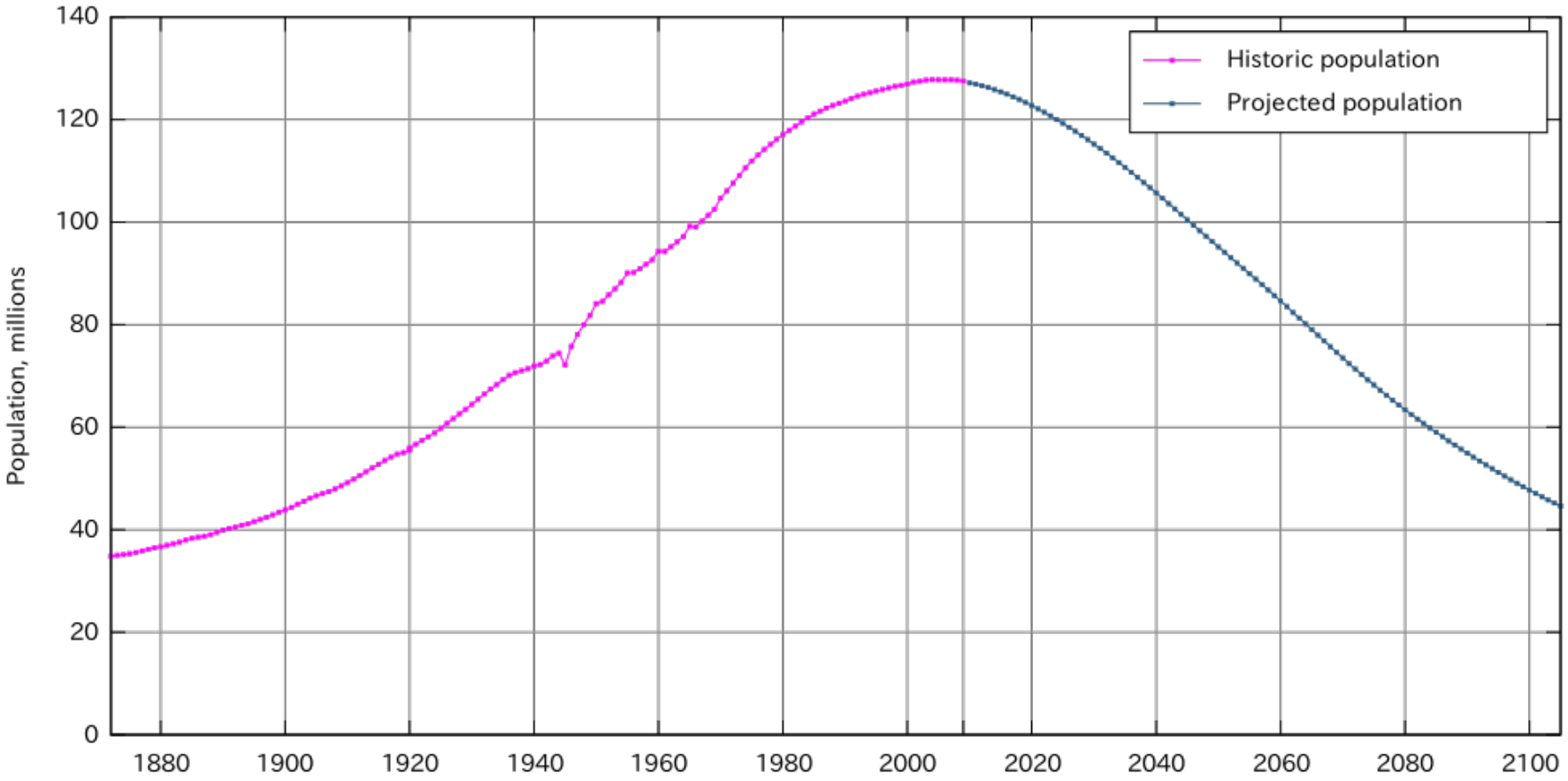

Population of Japan

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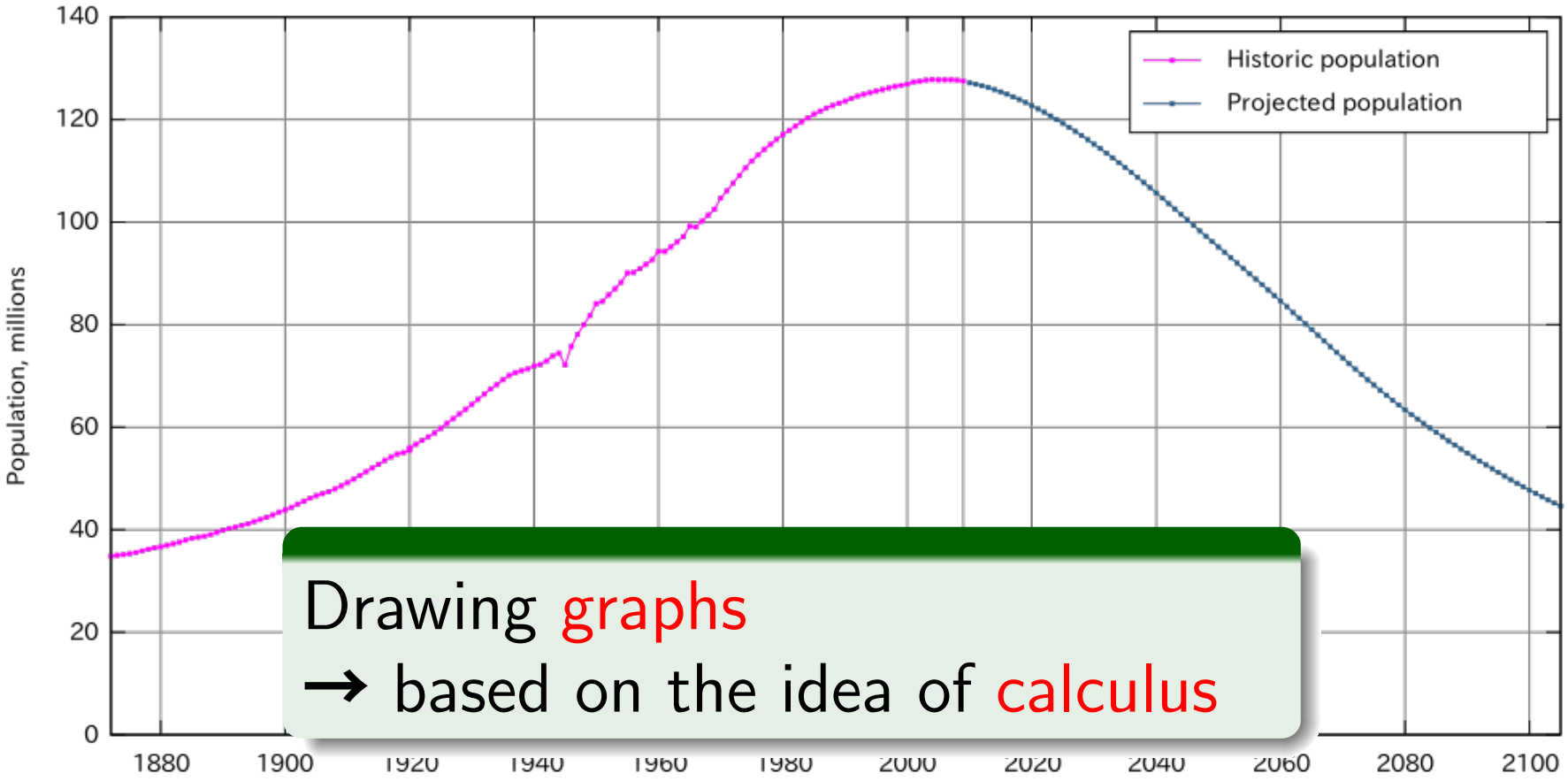

Drawing graphs
→ based on the idea of calculus

Population of Japan

Mathematics to describe “region”

Key words

“Local” and “Global”

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“Local” and “Global”

Local data and global data

- Integration
- Manifold (or Variety, *Stack*) – Local data and their relations determine global data.

Local and global

- Global – Observe the whole object.

Local and global

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- Local – only the neighborhood of a certain point.

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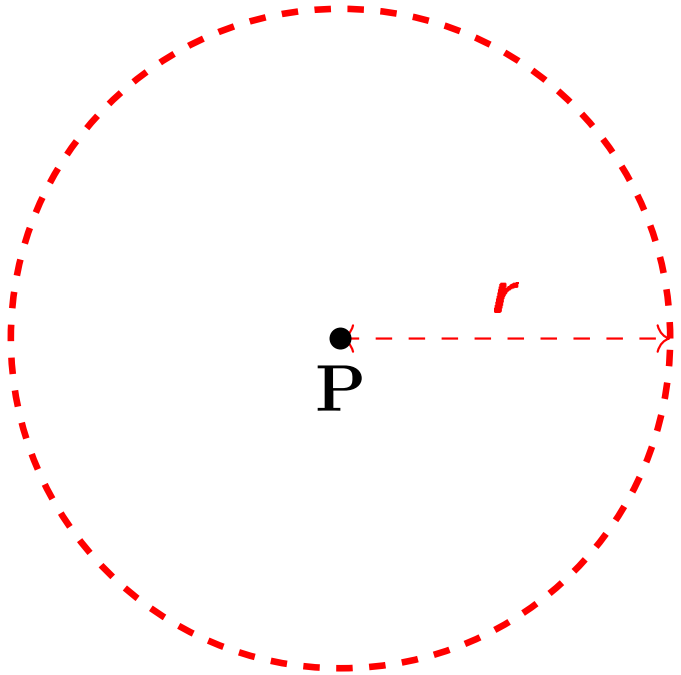

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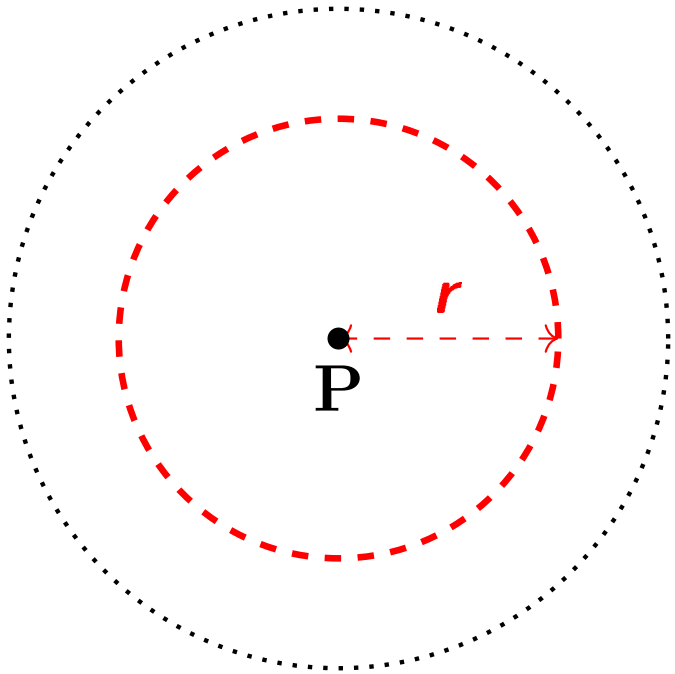

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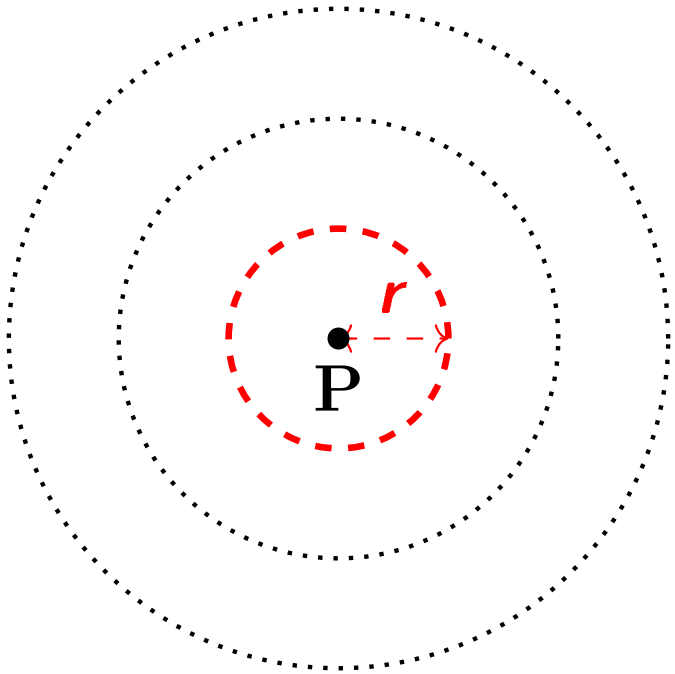

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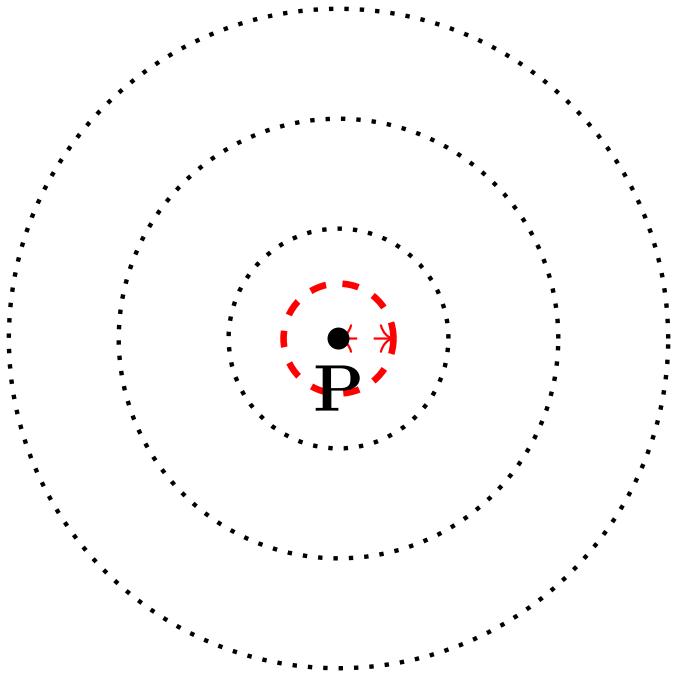
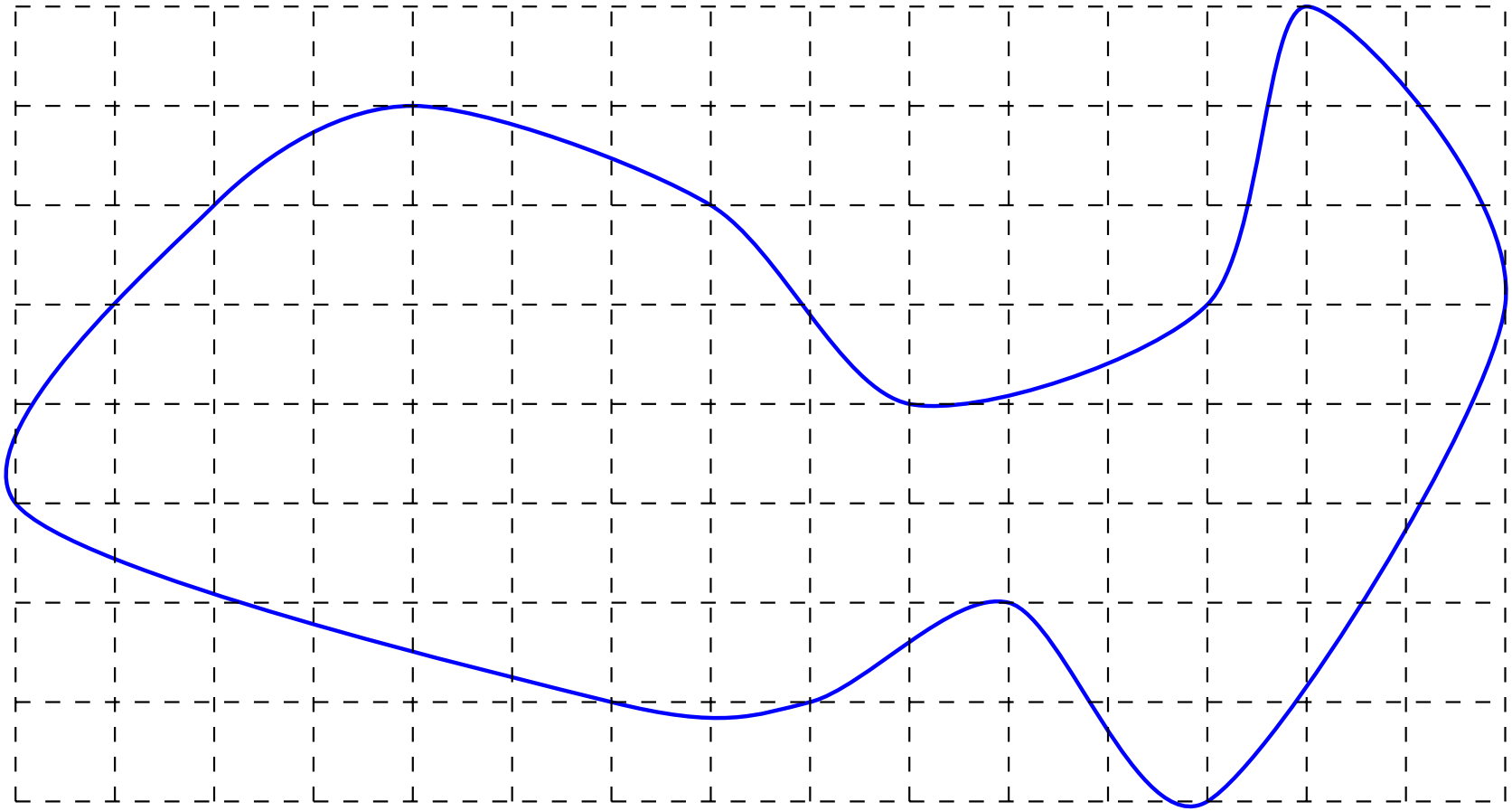


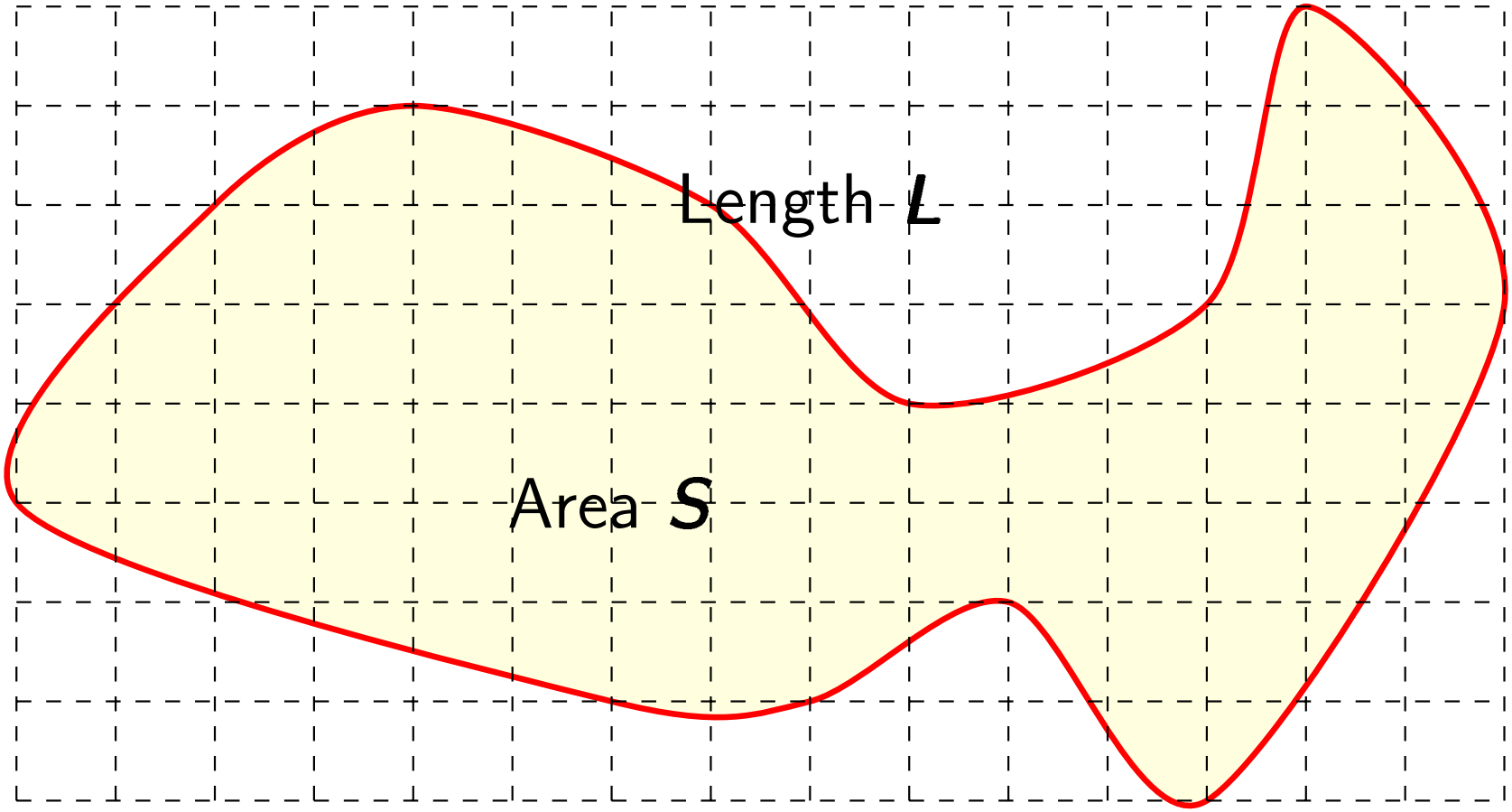
Example : local and global

Closed curve on a plane.



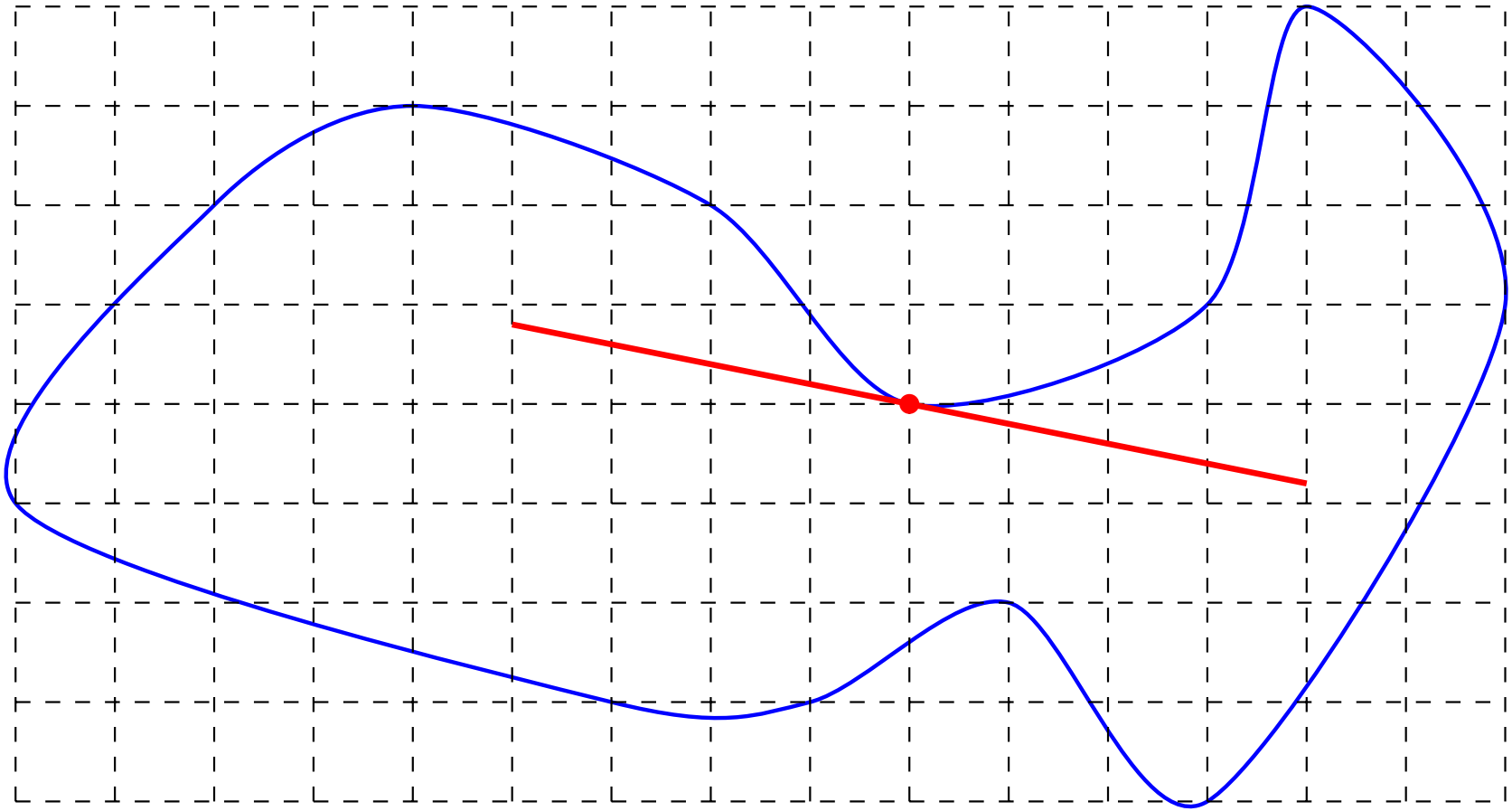
Example : local and global

Length, area — global data.



Example : local and global

Tangent lines, curveture — local data.



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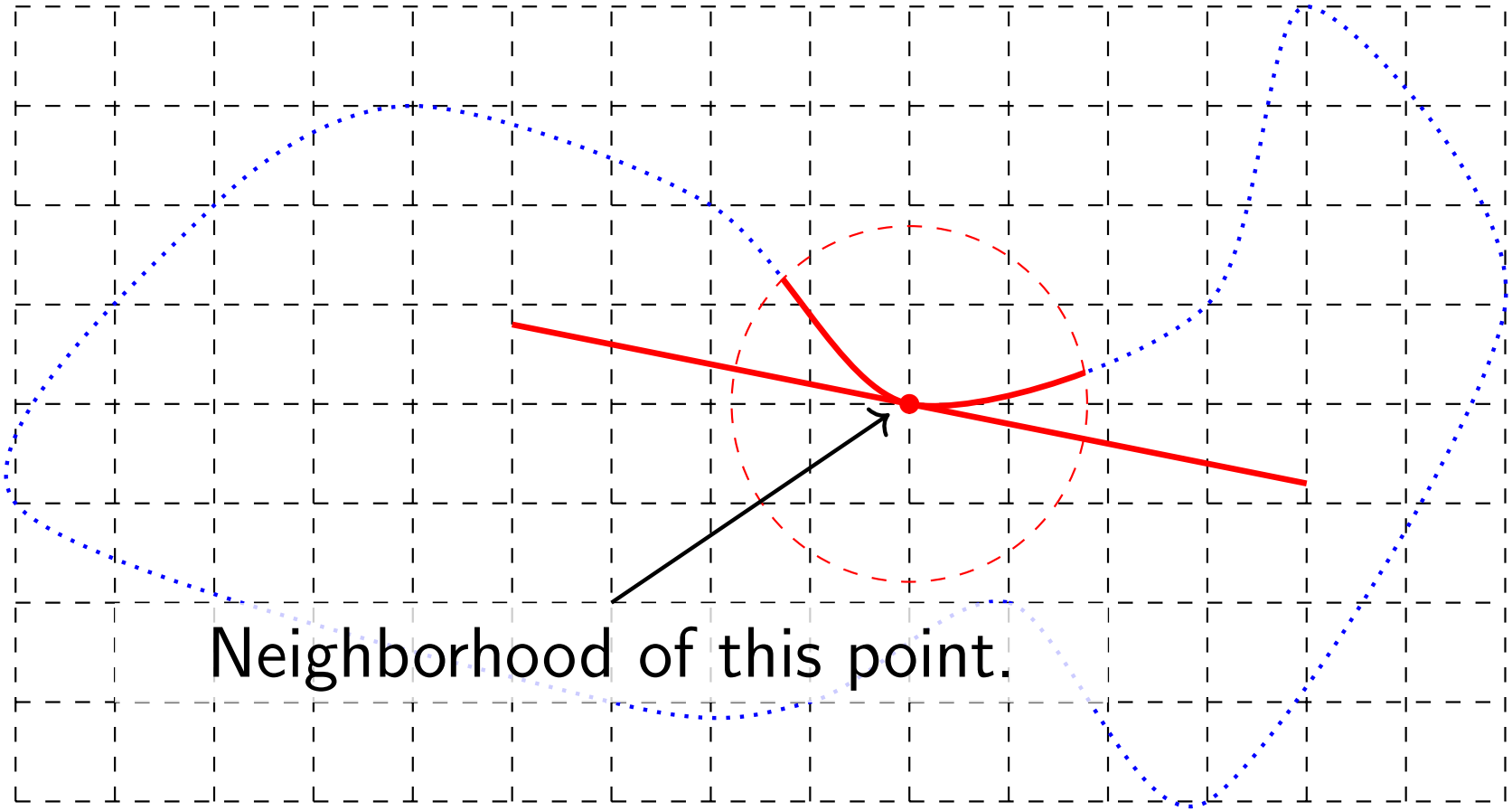

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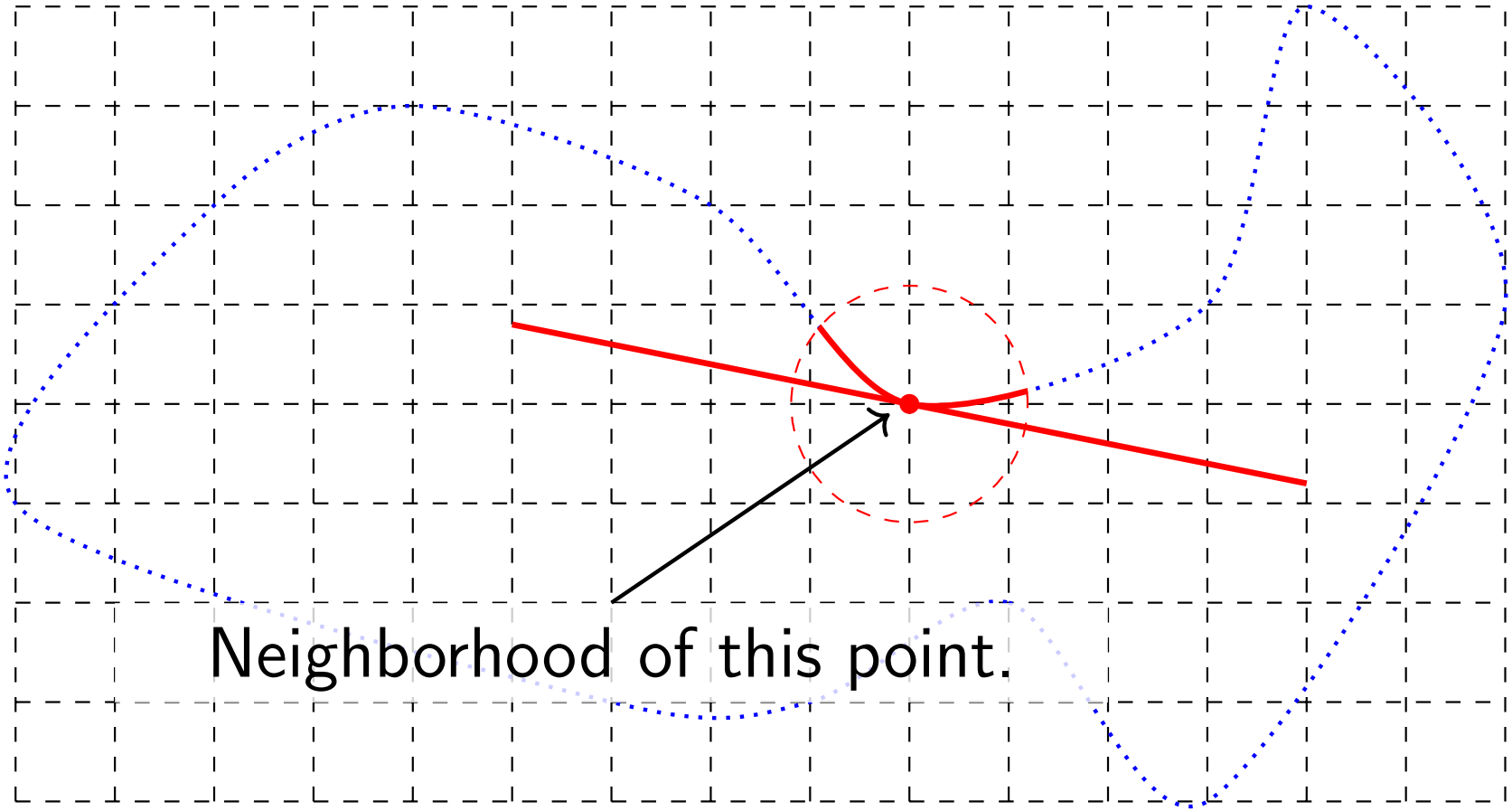

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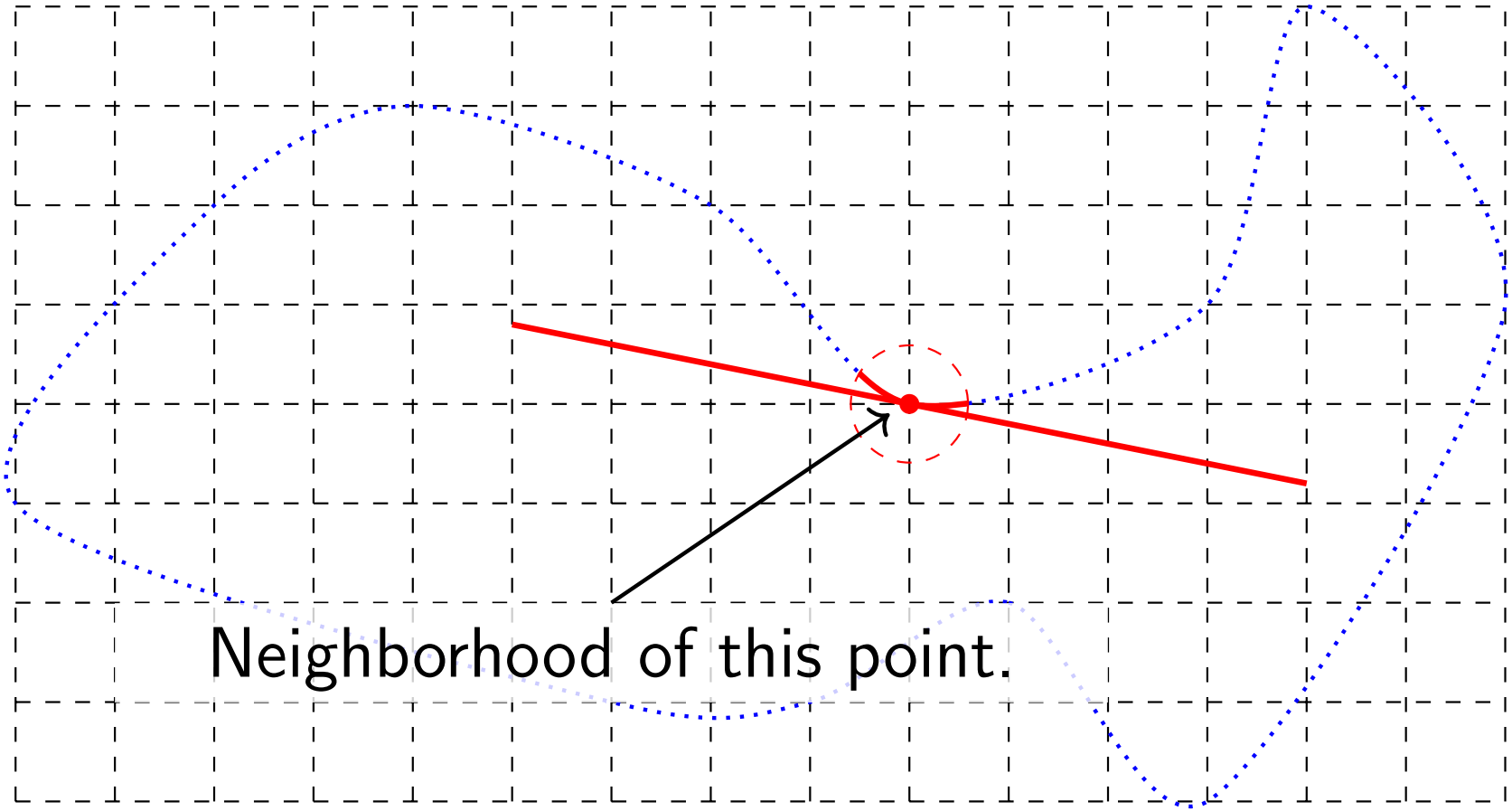
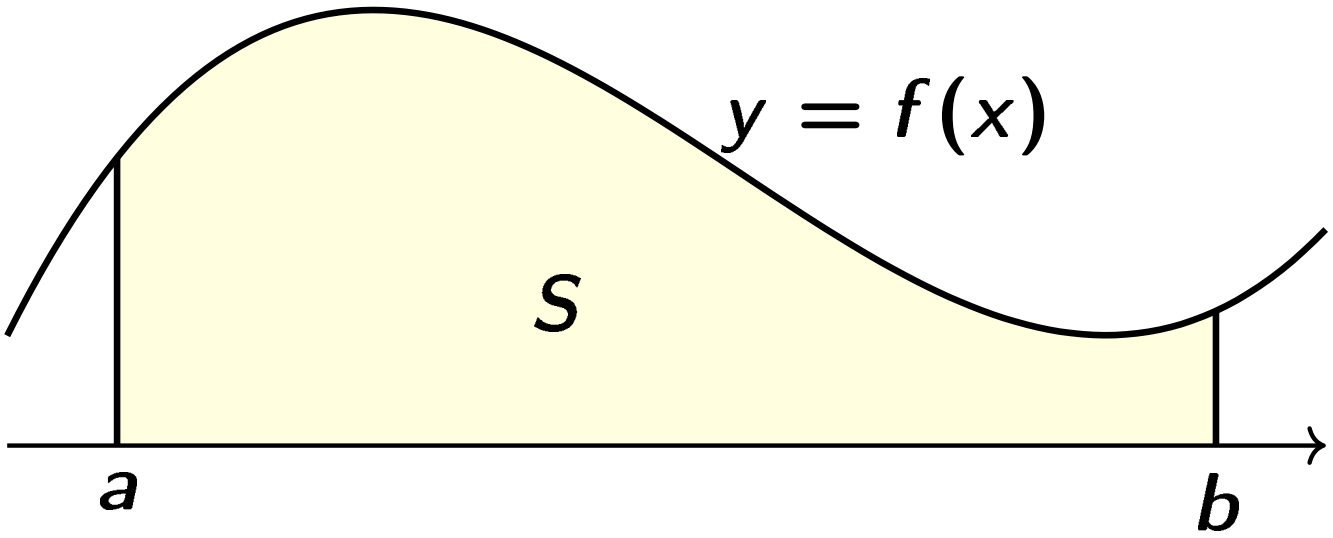
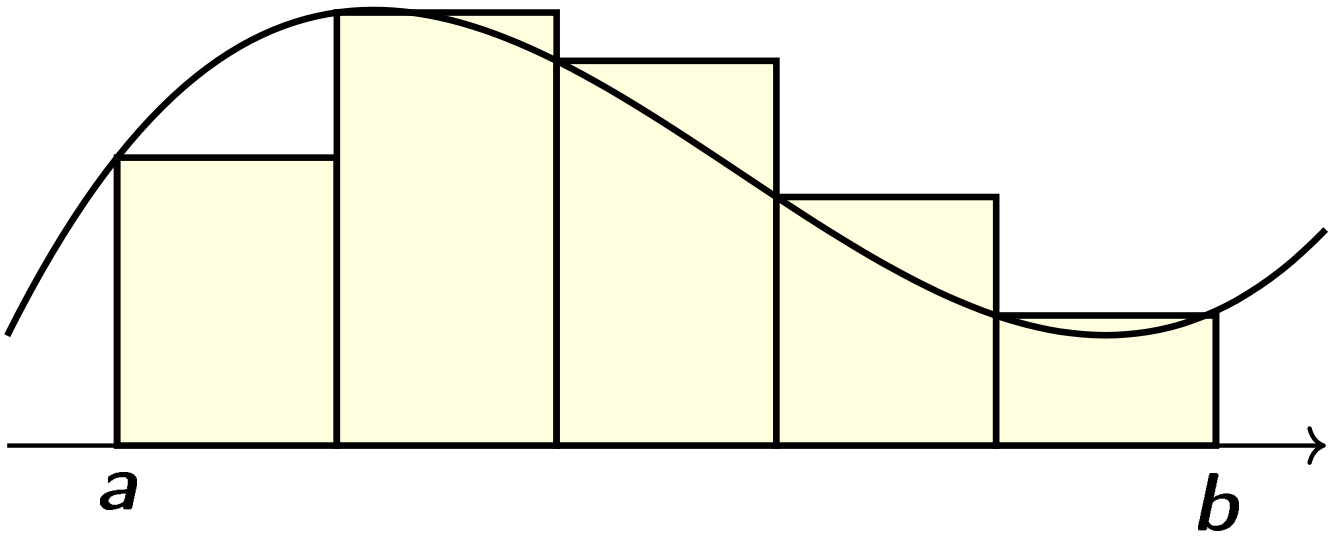


Integration



- Value of a function — local data.
- Area – global data.

Integration



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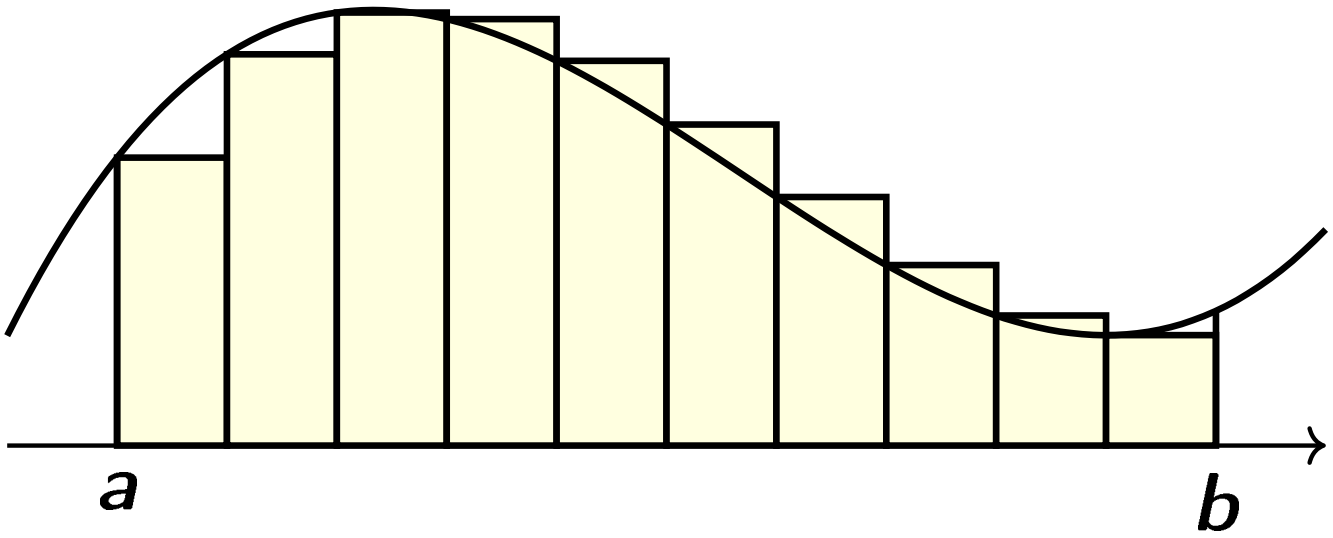

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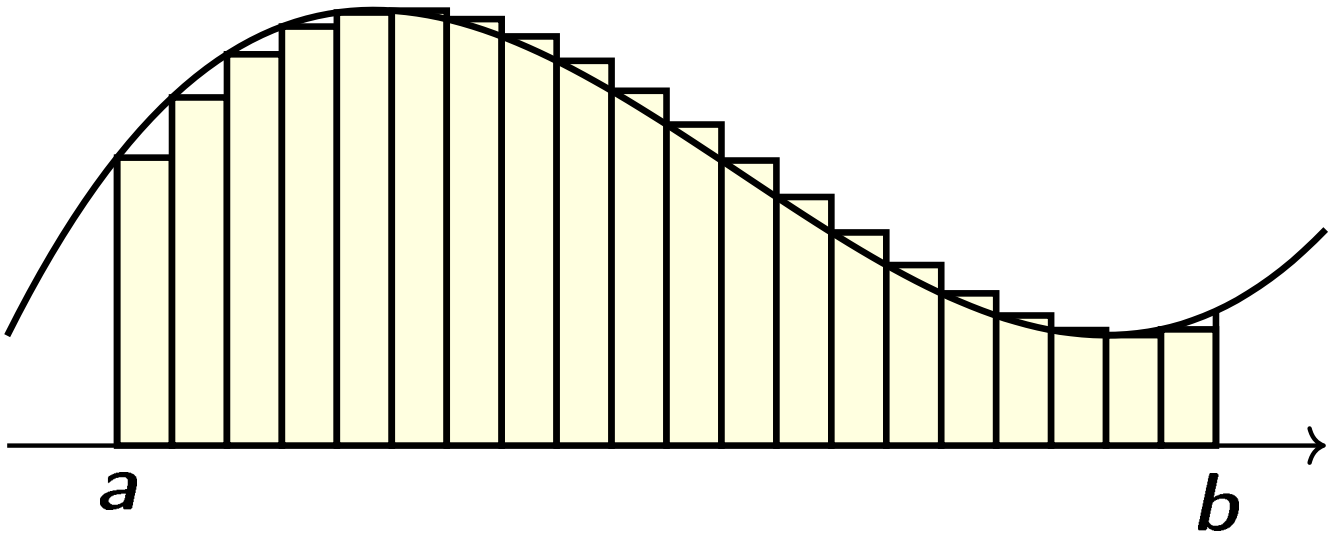

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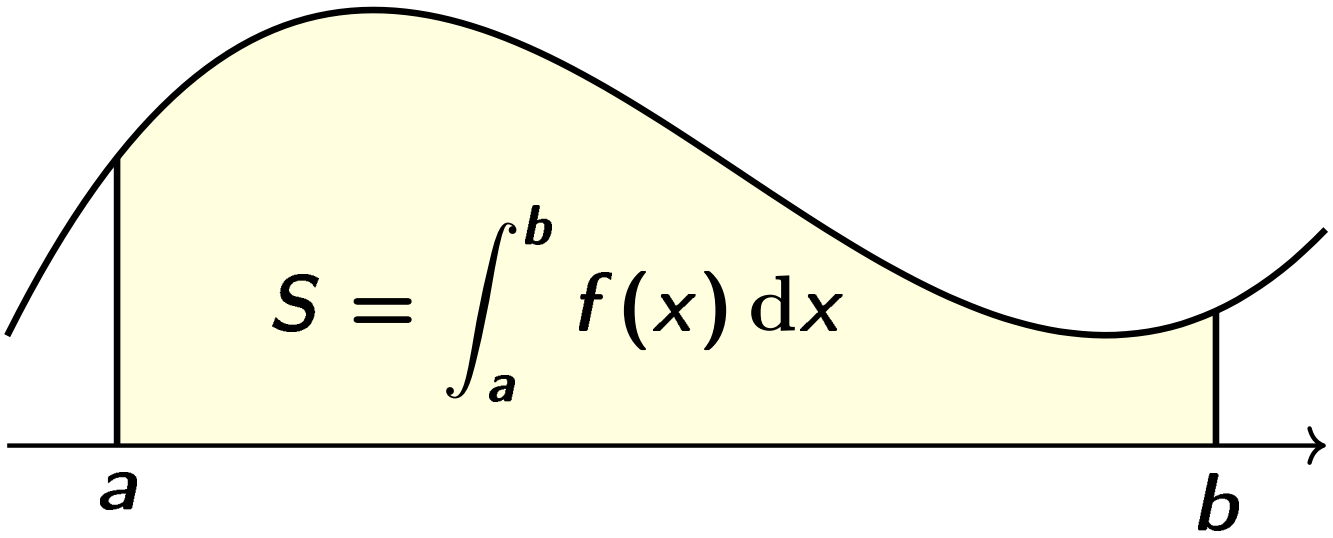

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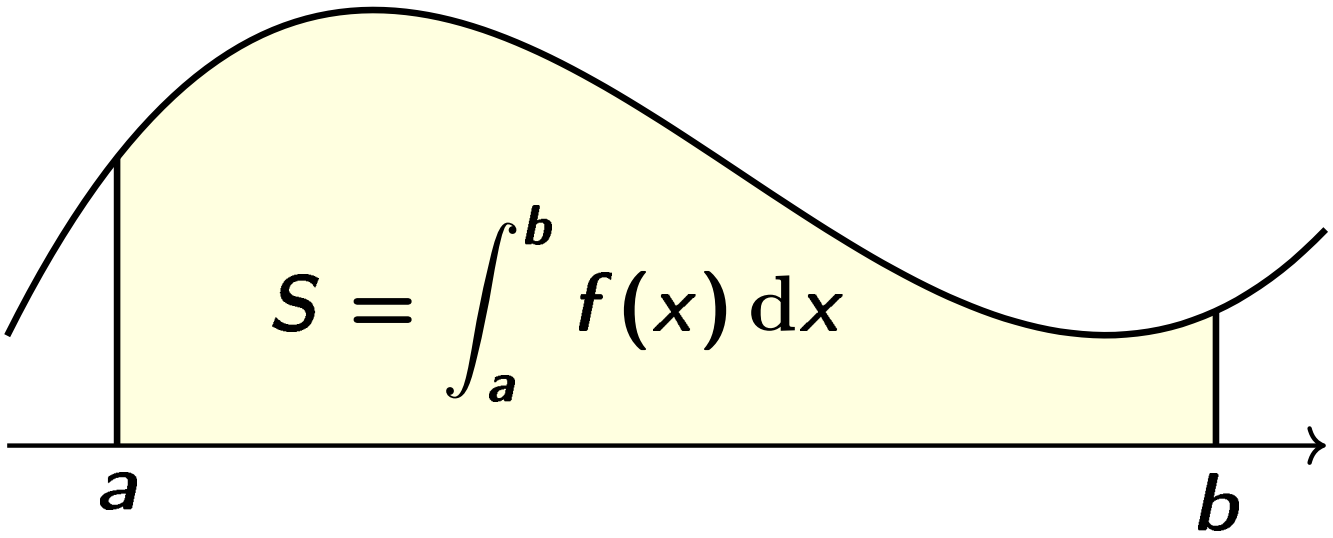
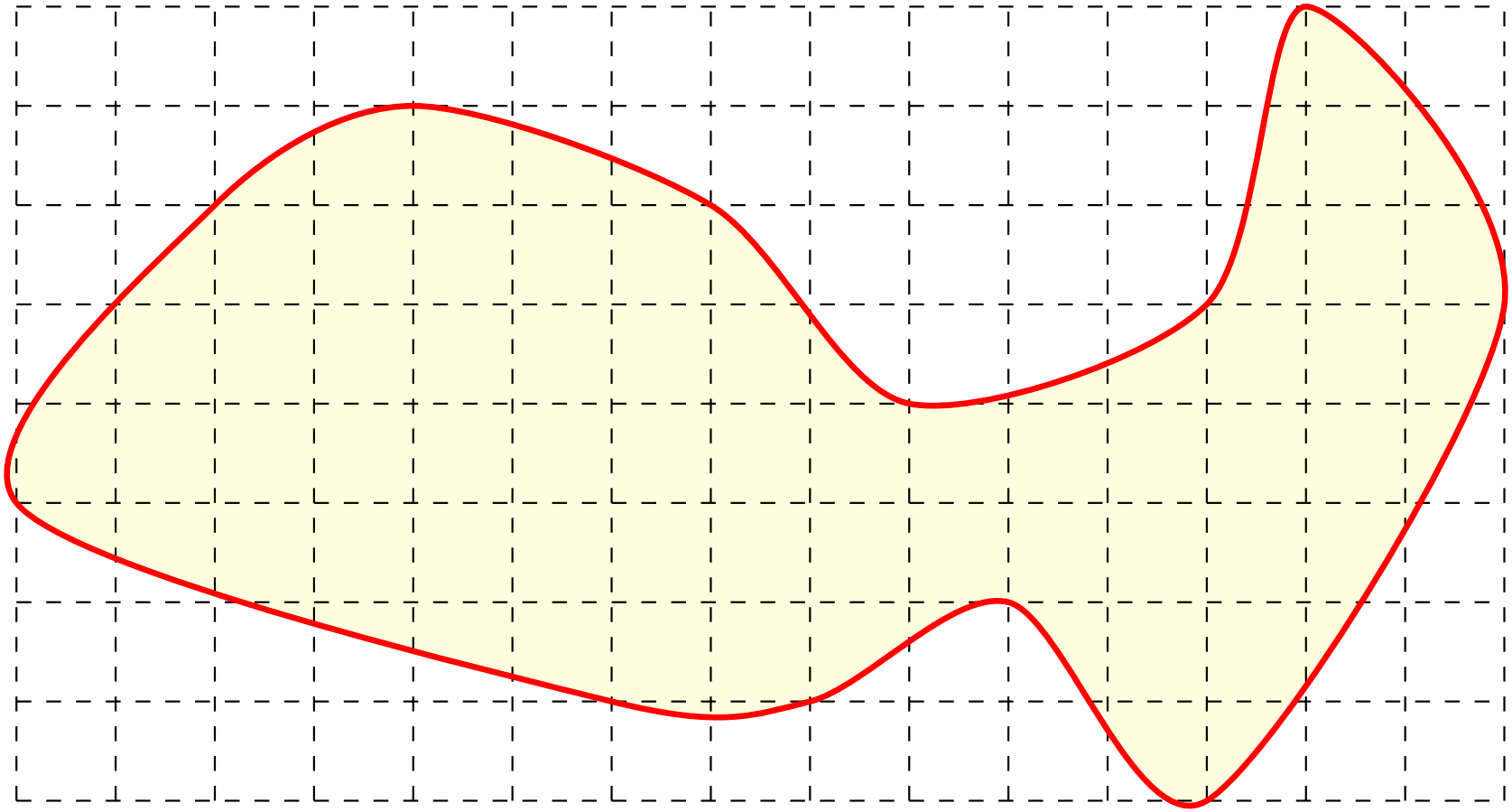


- Value of a function — local data.
- Area – global data.

Integration

Get global data from the collection of local data.

Length and area



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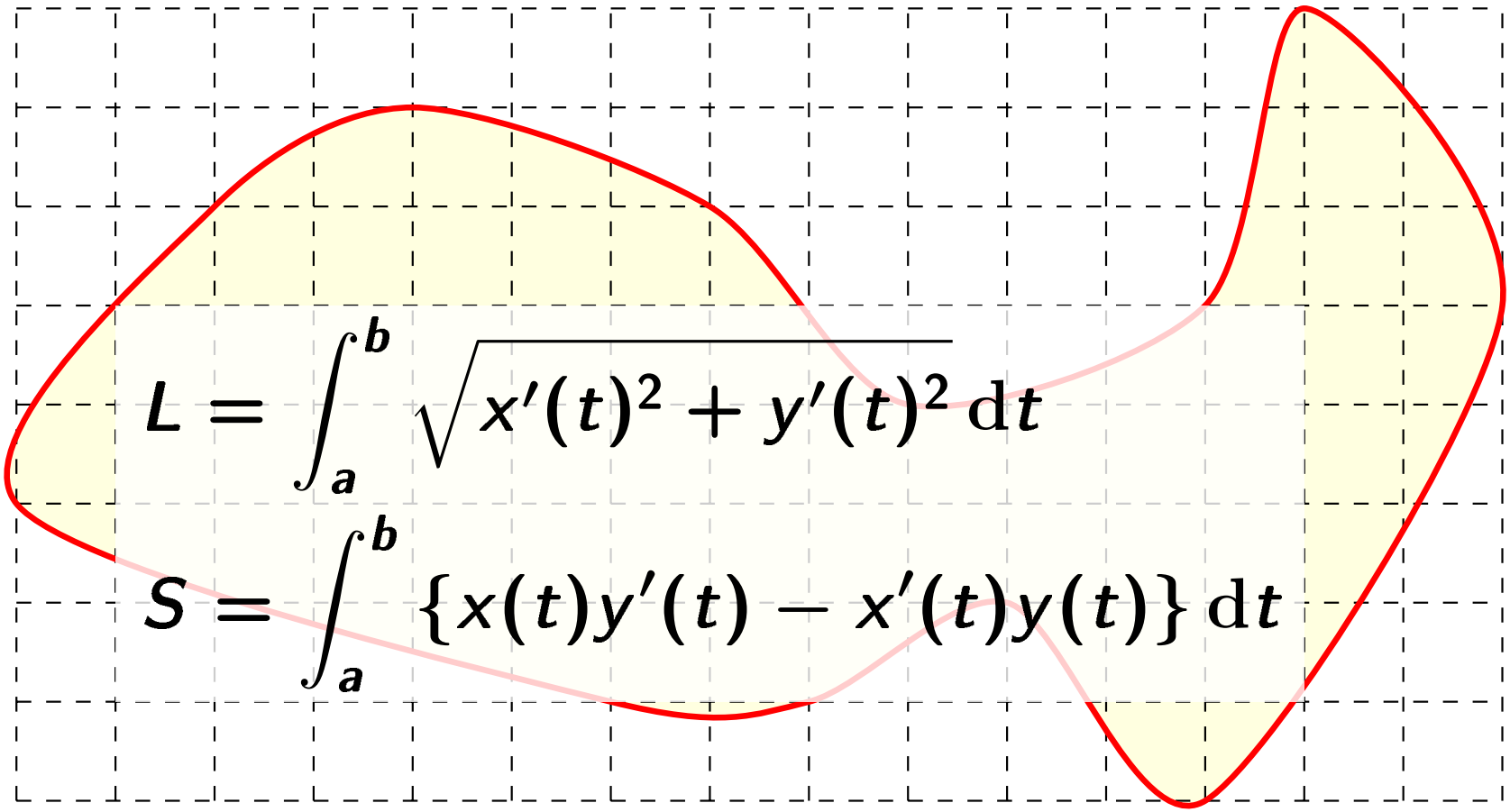

Manifold

Hard to grasp the whole.

- Cannot draw picture.
- Cannot measure or calculate.

Manifold

Hard to grasp the whole.

- Cannot draw picture.
- Cannot measure or calculate.

$\Rightarrow$ Easy to study locally.

- Set coordinates.
- Algebra and analysis work.

The Earth is ...

a sphere,

locally a plane.

The Earth is ...

Example : the Earth

The Earth (Globe) is a sphere (globe).
$\Rightarrow$ You may think earth is flat if you look locally.

The Earth is ...

Example : the Earth

The Earth (Globe) is a sphere (globe).
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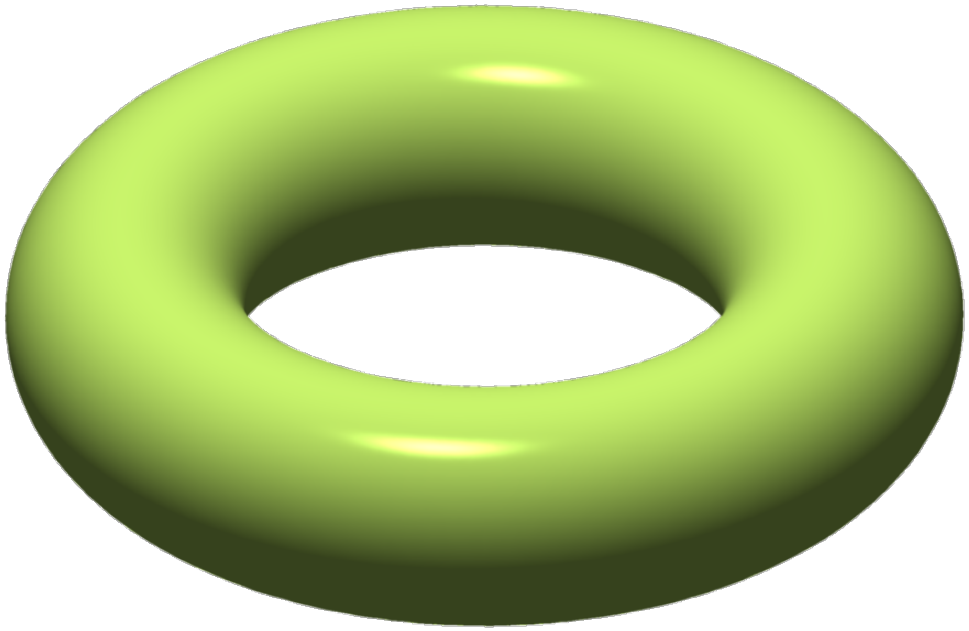
Locally approximate a sphere by a plane.

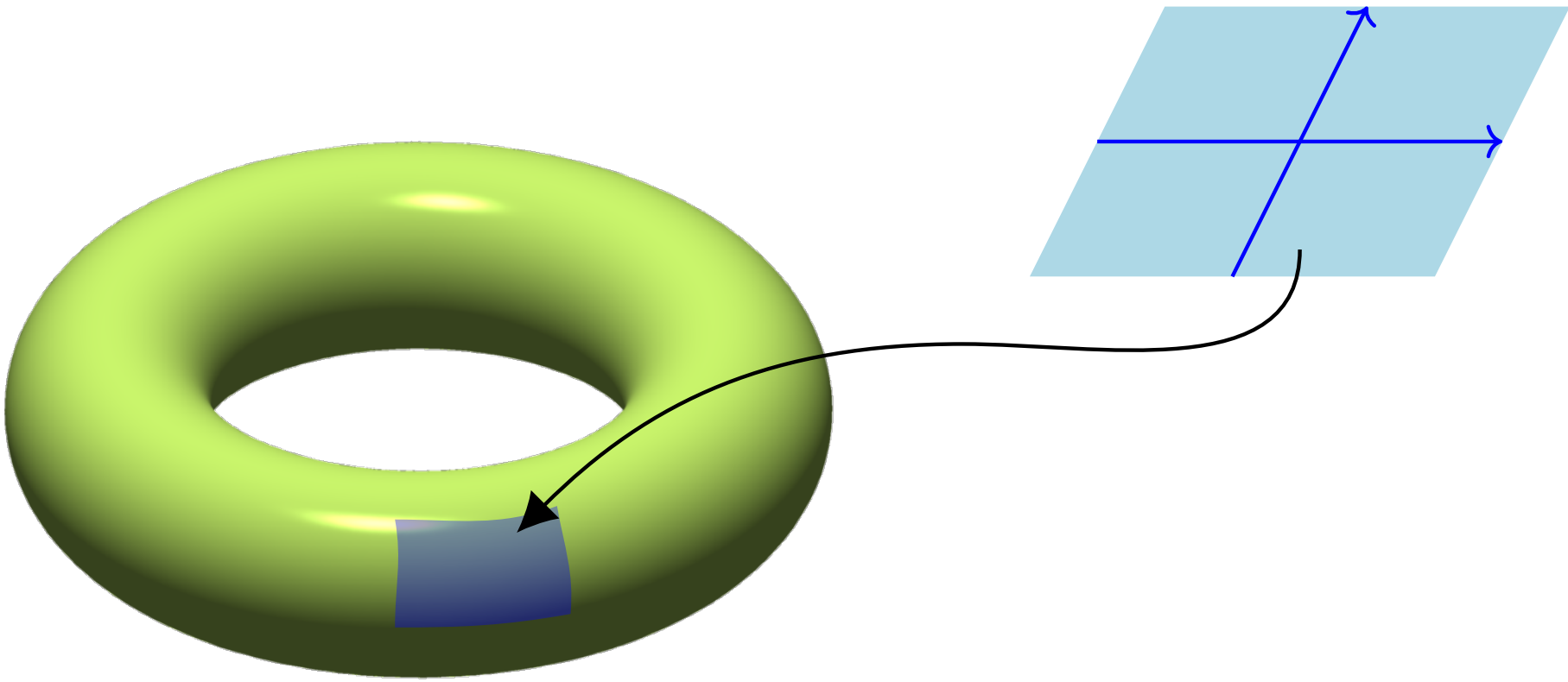
- Draw maps.
- Measure length and area on a plane.
- Use coordinates to point locations.

Manifold



Manifold

- Locally approximate by Euclidian spaces.



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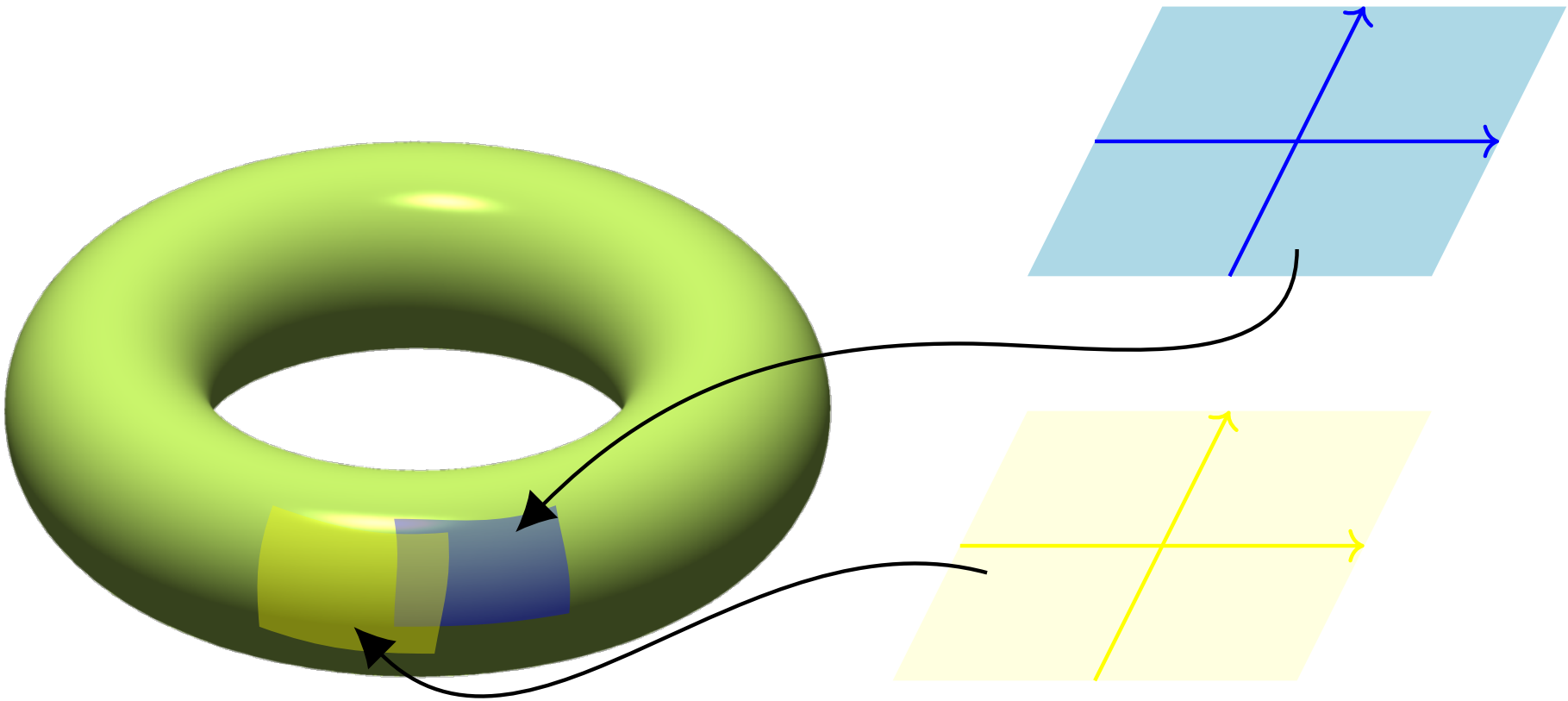

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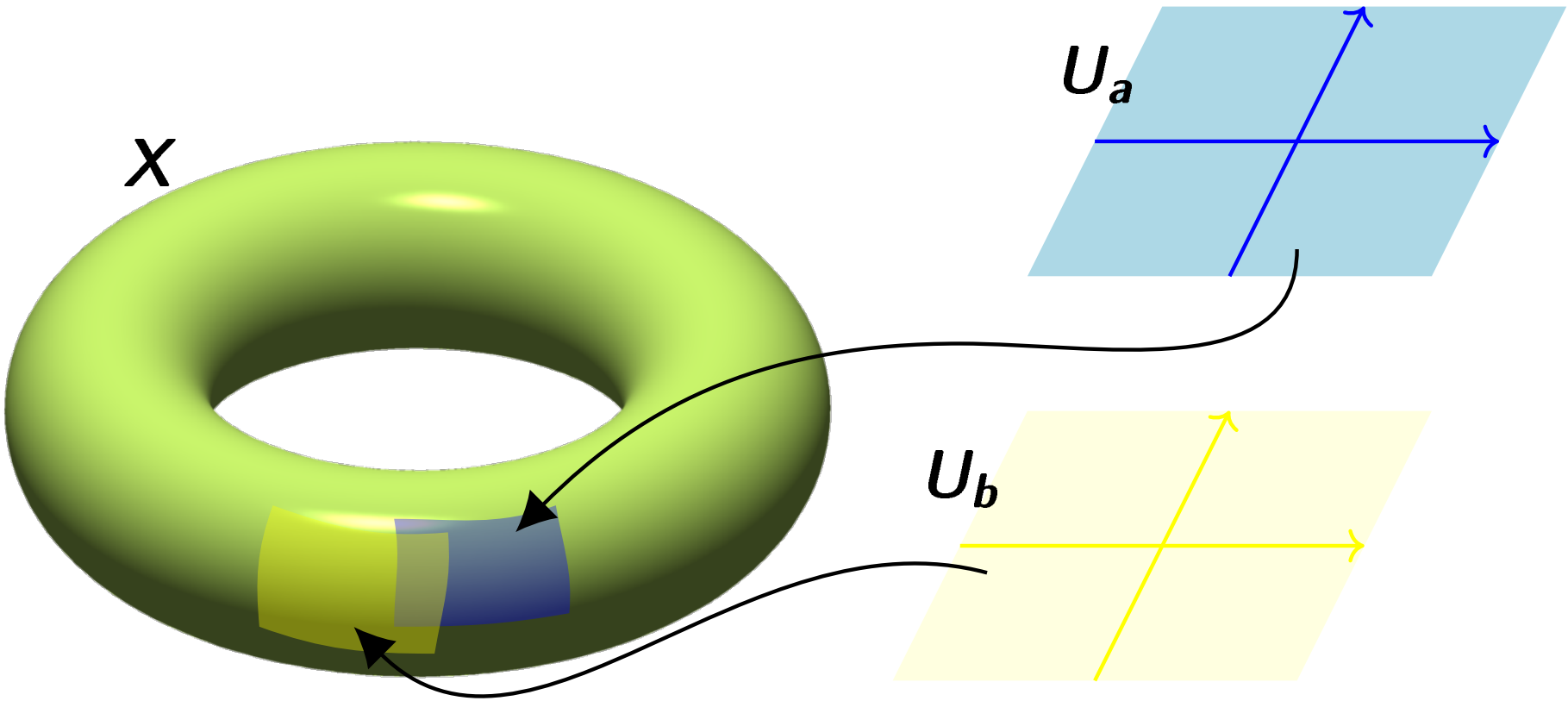
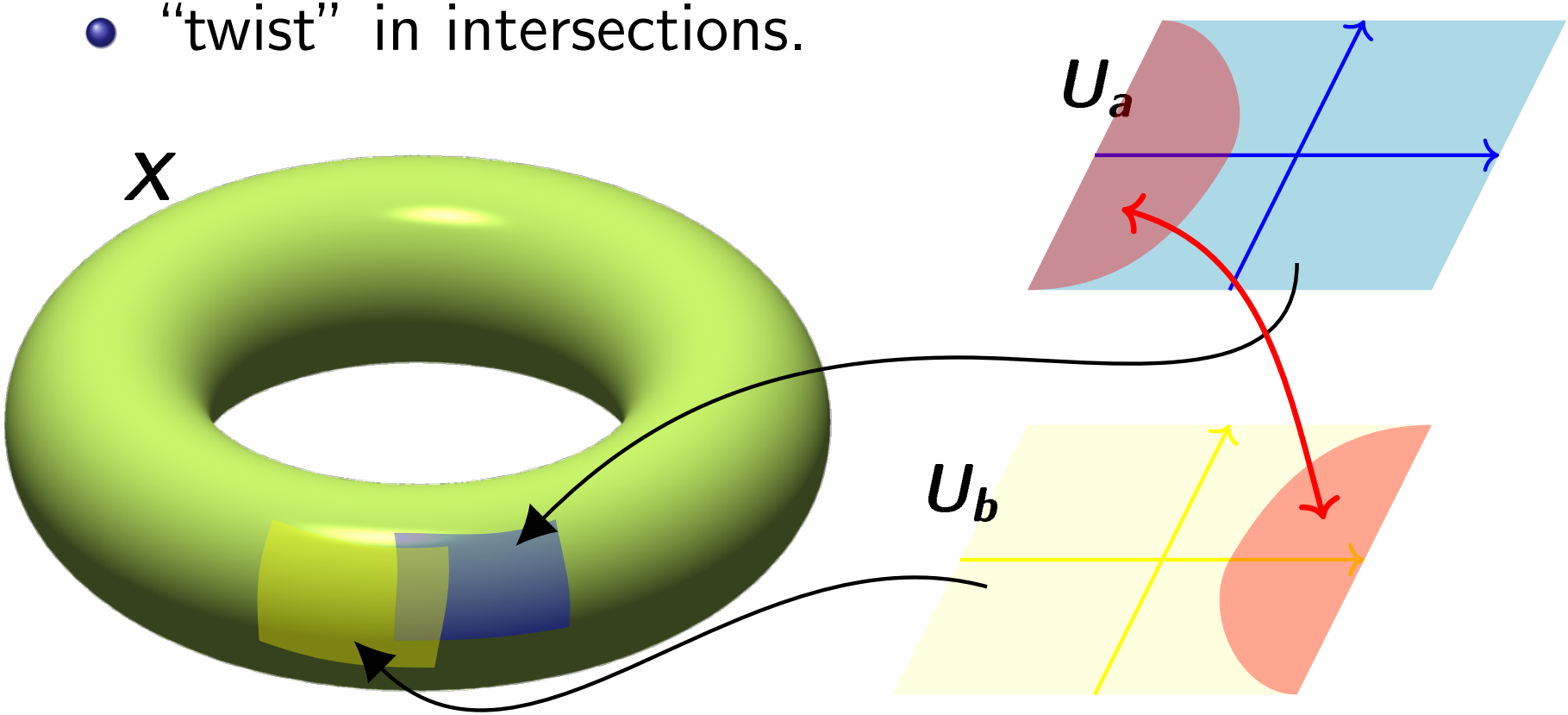


Manifold

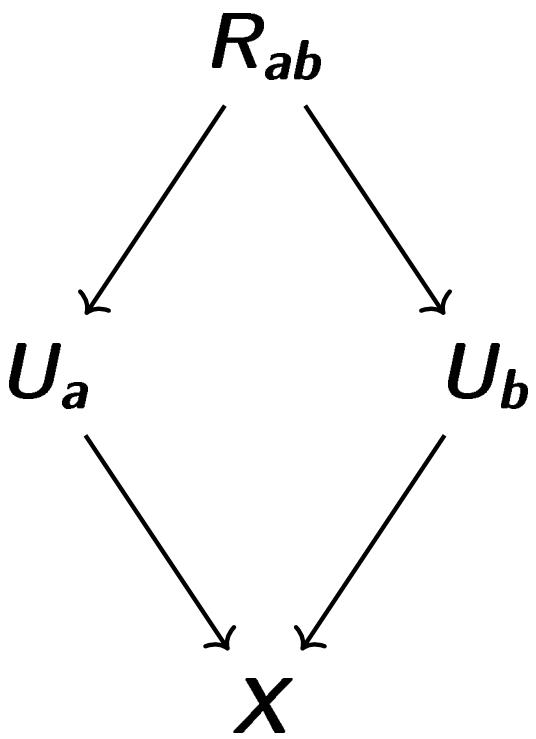
- Locally approximate by Euclidian spaces.
- "twist" in intersections.



Patching

Patching two maps requires twist.

Local approximation and patching

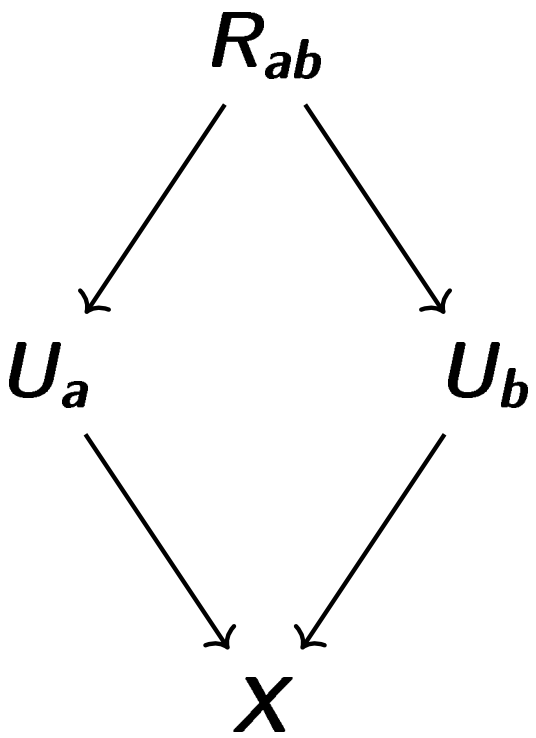


patching

local approximation

manifold

Local approximation and patching

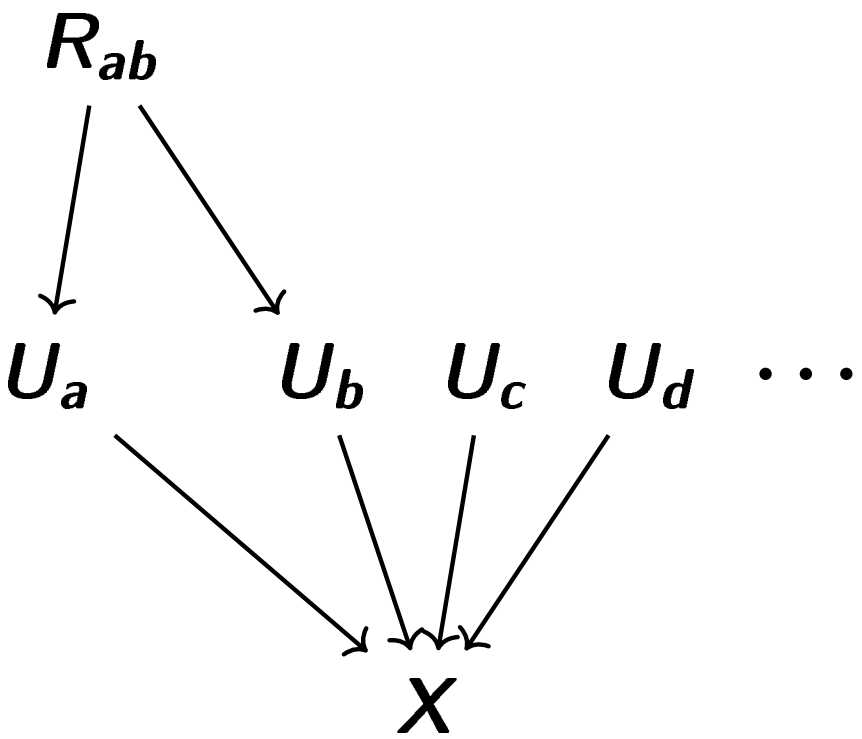


coordinate transformation

local coordinate

manifold

Local approximation and patching



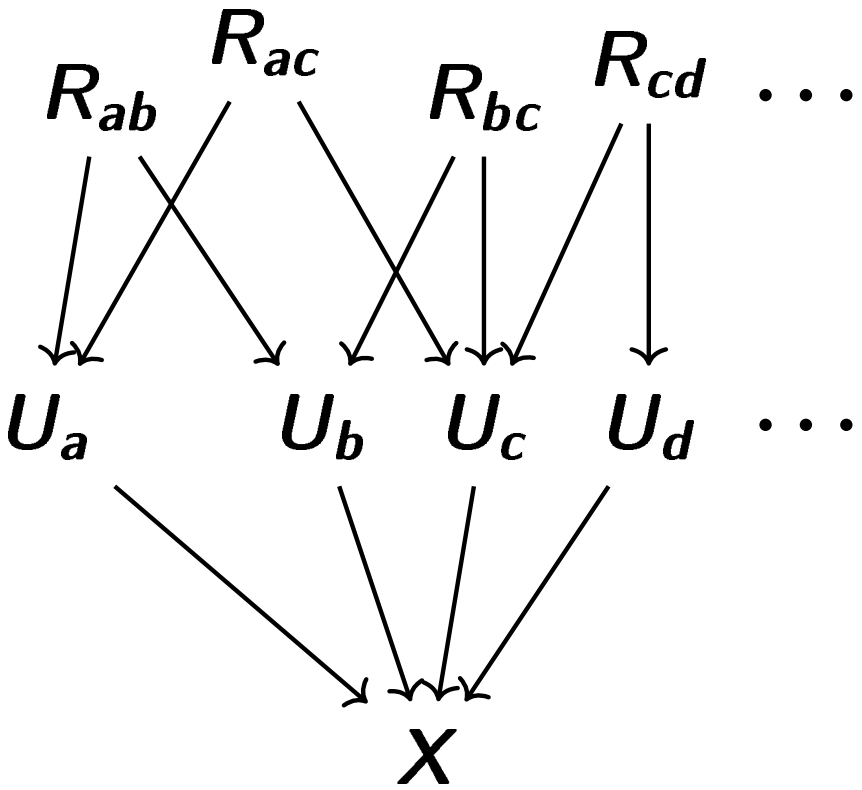
coordinate transformation

local coordinate system

— Cover the whole.

manifold

Local approximation and patching



coordinate transformations

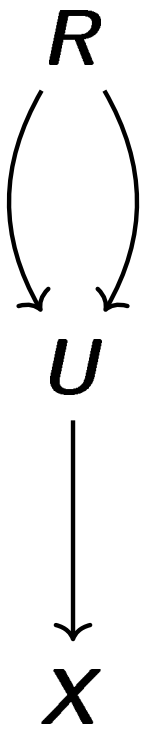
— For all intersections.

local coordinate system

— Cover the whole.

manifold

Local approximation and patching

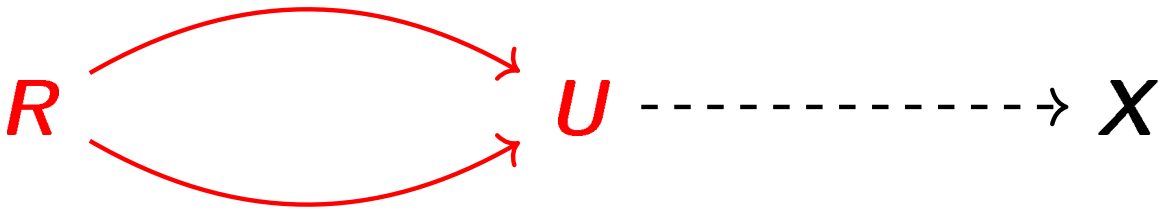


coordinate transformations

local coordinate system

manifold

Viewpoint of modern geometry



Principle

Local coordinate system and data of coordinate transformations detrmine the whole manifold.

Viewpoint of modern geometry

Thus we can use algebraic or analytic method to study manifolds, even if it no longer lies in a Euclidian space.

If we introduce more general concepts of “covering” and “patching”, we can treat much incredible objects such as differential orbifolds or algebraic stacks. We cannot draw pictures of them, but they are new “spaces” where mathematics works.

Viewpoint of modern geometry

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For example, "*Hom stacks*" in my thesis.

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